Answers to Sample Practice test for SHSAT GTSHSAT1 V1.0
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This sample test contains 60 assorted Mathematics questions. The actual exam contains 52 Mathematics multiple-choice questions and 5 grid-ins, as well as a multiple-choice ELA section. It is suggested that you take other practice exams before this one.

NOTE: Diagrams are not necessarily drawn to scale.
NOTE: \| mean square root, so \|4 means take the square root of 4.
NOTE: ^ means "to the power of" so 3^2 means 3 to the power of 2, yielding 9.
NOTE: .: mean therefore
NOTE: The actual exam only has 4 choices for multiple choice questions!
But for some questions in this sample test, I provide more choices.
I have also combined questions that might otherwise be separated.

MATHEMATICS

QUESTION 1. The "every 6 minutes" is constant for both Jack and Jill so it does not matter.

Givens:
Jack at 95th going lower 4 blocks per 6 min —> 95 - 4n
Jill at 20th going higher 1 block per 6 min —> 20 + n
n represents a 6 minute period

The math:
```
95 - 4n = 20 + n
    +4n        +4n
95      = 20 + 5n
-20       -20
75 = 5n
/5   /5
15 = n
```

This means it takes 15x6 = 90 minutes for them to crash into each other.
But that's not what the question asks.

But with n we can figure out where they crash:
95 - 4n = 95 - 4x15 = 95 - 60 = 35
Also:
20 + n = 20 + 15 = 35

35 = 35
So they crash on 35th street; the closest landmark is (B) Macy's

ALTERNATE SOLUTION USING "LOGIC":
From 95th Street to 20th Street is 75 blocks

If Jack is going at a rate of 4 blocks per 6 min and Jill is going at a rate of 1 block per 6 min then they are moving towards each other at a rate of 5 blocks per 6 minutes.

This means there are 75/5 = 15 6-minute-groups they run through together in total

Therefore Jack travels 15x4 = 60 blocks and Jill travels 15x1=15 blocks
If Jack left from 95th then he's at 35th, if Jill from 20 she's also at 35th.

ALTERNATE SOLUTION USING BRUTE FORCE:

```
Jack Jill
95   20th Street (minus 4 for Jack and add one for Jill)
91   21
87   22
83   23
79   22
```

. . . this becomes tedious so let's optimize it by having Jill go 10 blocks first which would mean Jack went 40 blocks:

```
Jack Jill
95   20th Street
55   30th now let's go back to by 1 for Jill
51   31
47   32
43   33
39   34
35   35
```

The rest is as above

We could have also tried 5 from the 55/30th point and gone straight to 35/35 that way too and whoop there it is still yielding 10+5 = 15

GRID-IN SOLUTION:
If it takes them 15 6-minute-groups then that is 90 minutes!
If they start running at 3pm they they crash at 4:30pm

QUESTION 2.
Givens:
100yds x 53 1/3 yds football field
Parent runs 16 ft/sec
Child runs Parent/4

"Direct" calculations upon givens:
Adding 10 yard end zones yields 120 yds x 53 1/3 yds football field
Child runs 4 ft/sec

Needed:
Calculate the length of the diagonal
Formula needed: c^2 = a^2 + b^2

Concern:
The field is in yards but the parent and child speeds involve feet
Convert feet to yards or yards to feet?
Well 16 and 4 seem poor in yards at least initially so let's leave them for now
So let's convert the field to feet:
120 yards is 360 feet
53 1/3 yards is 160 feet

So the field is 360ft x 160ft
Rip it through Pythag: c^2 = 360^2 + 160^2

360^2 = 129600 and 160^2 = 25600 .: c^2 = 155200
It might be a good time to learn how to get the square roots of numbers?
But we don’t need to as the problem tell us to round the diag to nearest 100
.: For our purposes only need to consider 300 vs 400 yielding 90000 vs 160000
.: 155200 is clearly closer to 160000 .: yields 400 is what we want

Alternately, let’s consider that we can factor 40 out of 360 and 160!
.: 360/40 = 9 and 160/40 = 4
.: c^2 = 9^2 + 4^2 = 81 + 16 = 97
Well \|97 is close to \|100 (Note: \| mean square root)
.: We can figure that’s about 9.8 or 9.9
.: If we scale the factor of 40 back into that, we get a number closing in on 400
.: The problem tells us to round the diagonal to the nearest 100
.: The diagonal is to be considered 400 ft

Now if the parent is running 16ft/sec and the child 4ft/sec
.: They are running towards each other at 20ft/sec
.: 400ft / 20ft/sec = 20 sec
The problem asks us to round to the nearest second but it already is!
.: Whoop there it is Choice B

QUESTION 3.
If we consider:

```
A+---+V
 |   |
 |  W+-----+X
 |         |
3|         |
 |         |
 |         |
 |         |
 |        Y+------------+Z
 |                      |
B+----------------------+C
             4
```

.: VW + XY + ZC must be 3
.: AV + WX + YZ = 4
.: This perimeter is 3 + 4 + 3 + 4 = 14

This is the same perimeter as if the shape were a rectangle, whereas the "staircase effect" has the same lengths and widths just "pushed in.”

In the problem, XY is broken with the added change involving D, E and F.

However note that EF is still allowing the height to stay at 3.
.: The space EF takes up is there either way whether broken off or part of XY
So even though the problems tells us that EF=1 it doesn’t matter.

What does change is that we add DE, and then after going from D to E and F it has to come back to the vertical line from F, which means the line going from F back to the hypothetical XY is added then as well.

.: if DE = 1.5 we ADDED 1.5 x 2 = 3

.: 14 + 3 = 17 Choice D

QUESTION 4.
a) Jack is slower than Kack

J < K

b) Kack is faster than Lack

L < K

c) Lack is faster than Mack and Nack

M < L
N < L

So what we have is:

J < K
L < K
M < L
N < L

i) Between Mack and Nack we don't know who is the fastest or slowest.

ii) Between Jack and Lack we don't know who is the fastest or slowest.

iii) In addition to (ii) we also do not know between Jack and Mack and Nack.

Iterating through the answer choices:
(A) The slowest runner must be Jack.
Jack might be the slowest, but Mack or Lack might be too.

(B) The slowest runner must be Mack.
Mack might be, but Nack might be too, furthermore, Jack might be too.

(C) The slowest runner must be Nack.
Nack might be, but Mack might be too, furthermore, Jack might be too.

(D) Either Mack or Nack must be the slowest runner.
Mack or Nack might be, but Jack might be too.

(E) Jack, Mack, or Nack could be the slowest runner.
Yup, fer shore. Another way to write the expression tree above in a kind of shorthand notation is:

((M&N) < L) & J < K

Note: & mean "and"

The "reverse" is true as well:

K > J & (L > (M&N))

Question:
Do you understand where Lack fits into things?
Can Lack be the slowest? No, see the hierarchy discussed/shown above.

QUESTION 5.
Lines (or curves) are concurrent if they intersect at a single point.

Lines PS, WR, and LQ intersect at point T (their point of concurrency). So the total number of concurrent lines is 3.

Points are collinear if they lie on the same line. So let's see:

* Points P, T, and S lie on line PS
* Points W, T, and R lie on line WR
* Points L, T, and Q lie on line LQ

.: That's 3 x 3 = 9 points

However, the question asks about unique points, and we counted point T 3 times

.: That's 7 unique points, even though all not on the same line

.: The answer is 3 + 7 = 10

QUESTION 6.

The multiplication does yield a repeating decimal:

$1.\overline{1} \times 1.\overline{1} = 1.\overline{2345678901}$

However, that value is none of the choice provided.

Also, choice (A), (B), and (C) are not mathematically supported by this multiplication, instead being illusory fabrications, or perhaps, wishful thinking.

Similarly, 1.21 ala (E) stinks just as much, as there is no basis to toss the repeating decimals away.

Process of Elimination, one often used test-taking strategy, points us to Choice (D) as the answer. But you should kick and scream to find answers and their rationale, so don't let (D) win by de facto fiat.

Note that $.\overline{1}$ is $\frac{1}{9}$, therefore $1.\overline{1}$ is $1\frac{1}{9} = \frac{10}{9}$

So the problem can be rewritten as $\frac{10}{9} \times \frac{10}{9} = \frac{100}{81} = 1\frac{19}{81}$

Oh yeah.

Food For Thought: The problem with hard questions is that they're hard. And the problem with easy questions is they're easy. Dwell there. This means everything it does and nothing it doesn't. Sometimes you just need to go deep.

More Food For Thought: When faced with the choices above, sometimes it makes sense to toss the outlier, in this case choice (D), I mean, "after all" it "couldn't" be the right answer. But sometimes too, if you give the outliers an extra look-see, they are just the thing you need to poke your internal numeracy radar.

Oh oh yeah.

QUESTION 7.
Let's label two more angles x' and y':

```
|
| y
|\
| \
|y'\
|   \
|    \
|     \
|      \
|    x' \  x
 ________\_____
```

This means x' : y' is 3 : 2, but it could also mean y’ : x’ is 3 : 2 but we don't care which way it goes so long as we can compute x' and y'

Well, there are 5 (3+2) angle parts altogether and x' + y' = 90
.: 90 / 5 = 18 .: 18 x 2 = 36 and 18 x 3 = 54 .: that's our two angles

And we can confirm that 36 + 54 = 90 and 54 : 36 is a 3 : 2 ratio

.: if y' is 36 then y = 180 − 36 = 144
.: if x' is 54 then x = 180 − 54 = 126
.: x + y = 126 + 144 = 270 Choice C

This would all still be true if x' was 36 and y' was 54

Furthermore, turns out that we don't need to compute the 3:2 ratio at all. Consider:

x' + y' = 90 .: y' = 90 − x' and x' = 90 − y'
y = 180 − y' = 180 −90 + x' .: y = 90 + x'
x = 180 − x' = 180 −90 + y' .: x = 90 + y'
Note: You should know these rules about angles such as x and y in addition to being able to compute them!

x + y = 90 + y’ + 90 + x’ = 180 + x' + y'
x' + y' = 90
.: x + y = 180 + 90 = 270   Choice C Word up!

Lastly, for a third perspective, as you know x + x’ is 180 degrees and also that y + y’ is 180 degrees, taken together (x + x’ + y + y’) they are 360 degrees. Since x’ + y’ is 90 then 360 − (x’ + y’) = 360 − 90 which is 270.

QUESTION 8.
There are 6 faces to a cube
.: 384 / 6 = 64 (this is the area per face)

Each face to a cube is a square
.: Each side length is \|64 = 8 (Note \| means square root)

.: Vcube = s^3 = 8^3 = 512

.: The volume of the cube is 512

.: The volume of a sphere of the SAME CAPACITY would also be 512 Choice A

QUESTION 9.

The ratio of WE to LR is \|3 : 1 or $\frac{\sqrt{3}}{1} = \sqrt{3}$

The ratio of VW to TR is similar, therefore we need to divide VW by \|3

.: We end up with $\frac{3}{\sqrt{3}}$ (Let's call this expression A)

How to get this in a “o\|i” form? Let's try to get rid of the denominator.

$\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \frac{\sqrt{3}}{1} = \sqrt{3}$ which is 1\|3 or 1 x \|3

.: o is 1, and i is 3 .: o x i = 1 x 3 = 3 Choice C

Alternate approach: Remember too that square root is the same as “to the power of 1/2”

.: In expression A above we have 3^1 divided by 3^(1/2)
When there is a common base division, we subtract the exponents

.: 3^(1-1/2) = 3^(1/2) = \|3 which as per above is 1 x \|3 or 1\|3

We can also solve this problem "directly" using a proportion:

$\frac{WE}{LR} = \frac{VW}{TR} \implies \frac{\sqrt{3}}{1} = \frac{3}{x}$

Run the cross product:

$x\sqrt{3} = 3$

$x = \frac{3}{\sqrt{3}}$

Which is what we had earlier to transform into o\|i form.

I used x here instead of o\|i for simplicity/convenience but I could have left it as is:

$o\sqrt{i} = \frac{3}{\sqrt{3}} = ... = \sqrt{3}$

Etc.

For another approach: some of you many also be aware of a 30-60-90 triangle which is a right triangle with angles 90 degrees, 60 degrees, and 30 degrees. Well, turn out it's "Pythagorean Triple" (PT) is 1-\|3-2

.: VWE is a 30-60-90 with a PT multiplied by a factor of \|3 hence \|3-3-2\|3

.: TRL is a 30-60-90 with no factor just straight up a 1-\|3-2 PT

.: TR is \|3 and the rest is the same as earlier.

QUESTION 10.
p represents the price of popcorn
d represents the price of drinks

“2 popcorns and 4 drinks cost $26” can be represented as:

a) $2p + 4d = 26$

“6 popcorns and 8 drinks cost $62” can be represented as:

b) $6p + 8d = 62$

We need to metabolize these two truisms against each other. One way is to "compare" these two equations against each other. Let's just put them next to each other for starters:

$2p + 4d = 26$
$6p + 8d = 62$

One way to solve this is to establish the same number of popcorns or the same number of drinks. For instance if we could get the first equation to involve 6 popcorns or 8 drinks then we could “interact it” with the second equation. If we were to pick targeting it for 8 drinks (we could have targeted it for 6 popcorns) we could do that by multiplying every term in (a) by 2:

$2(2p + 4d = 26)$
yields
a') $4p + 8d = 52$

Now if we list these two equations ((a’) and (b)):

$6p + 8d = 62$
$4p + 8d = 52$

we see that they both have an 8d. So let's subtract them:

```
  6p + 8d = 62
- 4p + 8d = 52
--------------
  2p  +0  = 10
```

That leaves us with $2p = 10$.: $p = 5$

If we plug $p = 5$ in both equations, we can see that we can get $d = 4$
To double check in the original equations:
$2p + 4d = 26$
$2x5 + 4x4 = 10 + 16 = 26$

$6p + 8d = 62$
$5x6 + 8x4 = 30 + 32 = 62$

The question asks: how much is 6 popcorns and 12 drinks? That is represented as:

$6p + 12d = x$

Since we now know p and d, we get:

$6x5 + 12x4 = 30 + 48 = \$78$

We solved this by making the coefficient of d in each equation the same and solving for p. We could have also solved by making p have the same coefficient in both equations:

$3(2p + 4d = 26)$
yields
$6p + 12d = 78$

Subtracting we get:

```
  6p + 12d = 78
- 6p +  8d = 62
--------------
        4d = 16 .: d = 4
```

And the rest works as previous, the only difference here is that we solved for d and earlier we solved for p.

This can work via division too; for instance we could divide (b) by 2 yielding:
b’) $3p + 4d = 31$
Subtracting (a) from (b’) yields $p = 5$

For a different solution approach, we could have also taken either (a) or (b), wrote out what p= or d= equals and substituted that into the other equation. So again if these are our equations:

a) $2p + 4d = 26$
b) $6p + 8d = 62$

Let's pick solving for p in (a) and then using that to solve for d in (b):

$2p + 4d = 26$
$2p = 26 - 4d$
$p = 13 - 2d$

Now substitute in (b), remembering to honor order of evaluation (“PEMDAS”):

$6p + 8d = 62$
$6(13 - 2d) + 8d = 62$
$78 - 12d + 8d = 62$
$78 - 4d = 62$
$78 - 62 = 4d$
$16 = 4d$
$4 = d$

And again all as before. Instead of solving for p in (a) and substituting to solve for d in (b) as just done above, we could have also solved for d in (a) and substituted to solve for p in (b), as well as, solving for p in (b) and substituting to solve for d in (a), and solving for d in (b) and substituting to solve for p in (a). (That’s a mouthful but spend a moment to think it through.)

This all said (whew!!), there is still another way so solve this. Please note that if you go back to the question, it tells us:

c) 2 popcorns and 4 drinks cost $26

And asks us how much is

d) 6 popcorns and 12 drinks

If you note (d), it contains 3 times the amount of each item used in (c) .: the cost must be 3 times too!

.: 26x3 = $78 🎉 ☮

As a side side-note you also know that since 6 popcorns and 8 drinks cost $62 then 6 popcorns and 12 drinks must be greater than $62 (by 4 drinks worth). It’s interesting how much we can sometimes determine from so little!

QUESTION 11.
Let's make sure you're aware of exponent rules for starters. That is to say:

$$X^{-y} = \frac{1}{X^{y}}$$

In English: a base to a negative exponent (-y is the negative exponent in our example) is the same as 1 divided by the base to a positive value of the exponent used (in our example +y, or just y).

We will also be making use of the rule that an entity divided by a fraction is the same as that entity being multiplied by the reciprocal of the fraction in question.

Given these two rules, as with many problems, there are a few ways to approach this.

Let's start by exploring the denominator from the question:

$$\frac{1}{X^{-2}}$$

Given this fraction and the exponent identity stated above, this is the same as:

$$\frac{1}{\frac{1}{X^{2}}}$$

which when we follow the reciprocal rule is the same as X^{2}, try it! 1 divided by that fraction is the same as 1 times that fraction's reciprocal (x-squared) which is x-squared.

This means we can replace that into the original problem, yielding:

$$\frac{X^{-2}}{X^{2}}$$

The numerator X^{-2} is the same as $\frac{1}{X^{2}}$

Therefore in effect we have:

$$\frac{1}{X^{2}} \times \frac{1}{X^{2}} = \frac{1}{X^{4}}$$

There are many ways to combine the division and exponent rules and end up with this same fraction. Try them!

If X=2 then $X^{4} = 2^{4} = 16$

Therefore we get 1/16. However, the question asks us to solve for the reciprocal, hence 16 Choice G. Note: the current SHSAT only provides 4 answer choices, not 13 as as I did here.

As well, to solve this you could have replaced X from the get-go and not at the end, which still needs to uphold the exponent rules discussed above:

$$.:\ 2^{-2} = \frac{1}{2^2} = 1/4$$

$$.:\ \frac{1}{2^{-2}} \quad \text{must be} = 4/1 = 4$$

.: We have 1/4 divided by 4 which is the same as 1/4 x 1/4 = 1/16
.: The reciprocal of 1/16 = 16

Additionally it's possible to use decimals, but that doesn't turn out to help us in this problem as all the answers are either whole numbers or fractions and therefore there is no pressing need to use say 0.25 instead of 1/4, etc. If such a need is a consideration, by all means use decimals.

This problem may seem like it’s only involving useless tricks or trivial math, but it is emphasizing aspects of exponent math which will become useful as you explore algebra, polynomials, etc. further, and math you must get right.

QUESTION 12.
If
f represents the first mixture
s represents the second mixture

then
f + s = 60
30%f + 15%s = 20% x 60

Let's normalize these together.

One way to do that is with substitution, as from the first equation we can also get:
f = 60 - s   and
s = 60 - f

So let's choose to substitute f into the second equation:

30%(60 - s) + 15%s = 20% x 60

which means that we have effectively removed f and now only need to solve for s:

30% x 60 - 30%s + 15%s = 20% x 60
30% x 60 - 15%s = 20% x 60
30% x 60 - 20% x 60 = 15%s
10% x 60 = 15%s
s = 40
.: f = 20

Feeding those values into the second equation to double check we get:

30%f + 15%s = 20% x 60
30% x 20 + 15% x 40 = 20% x 60
6   + 6   = 12
12 = 12

That is solving for f and s using substitution. We could also solve this with simultaneous equations whereas we balance the two equations against each other. We do this by trying to remove one of the variables by normalizing the one equation against the other.

First, since every term of the second equation is divided by 100 will just x 100 on each term for convenience effectively throwing it away, yielding:

30f + 15s = 20 x 60 = 1200

Since the first equation is

f + s = 60

then we look to cancel out the f's or the s’s. In this case either will work. We can do this by multiplying each term in the first equation by 30 or 15 depending upon whether we want to knock out f or s. So let's use 30:

30(f + s = 60)
30f + 30s = 1800

Now we just subtract the two equations:

```
   30f + 30s = 1800
-  30f + 15s = 1200
-------------------
    0  + 15s = 600
```

If 15s = 600 .: s = 40

QUESTION 13.
The sum of the known scores are:
90 + 80 + 98 = 268

The total possible for all 5 tests would be:
92 x 5 = 460

Therefore the known scores plus the 2 others test can be represented as:
268 + 2x = 460

Solving:
2x = 460 – 268 = 192
x = 96   Choice C

Double checking:

```
90 + 80 + 98 + 96 + 96   460
---------------------- = --- = 92
           5              5
```

QUESTION 14.
If a cube just fits perfectly it means the height of the cylinder must be the same as the height of the cube.

.: The height of the cylinder is also 8.

If a cube just fits perfectly, it means each vertex of the cube meets the edge where the cylinder's base meets the the cylinder's curved surface – in other words where the vertexes of the square would meet the circumference of the base, therefore, at 4 points on the “top” and 4 on the “bottom” of the cylinder.

Looking down from the top, this looks as a square inscribed in a circle, in our case fitting into/filling out the cylinder's face perfectly and both having the same center.

Under this situation, the length of the diameter of the circle is the same as the length of the diagonal of the square.

We can use the Pythagorean Theorem to calculate the diagonal as we know the length of the side of the square is 8.

```
.: c^2 = a^2 + b^2
       = 8^2 + 8^2
       = 64 + 64
       = 128
    \|(c^2) = \|128    (Note: \| means square root)
    c = \|128
```

Let's simplify \|128:

```
   \|128 = \|64 x 2 = \|64 x \|2 = 8\|2
   (this kind of unravels what we just did to compute c, but so be it)
```

Note: If you knew your Pythagorean Triples, you could have also noticed that the square with the diagonal through it inscribed in the circle establishes an isosceles triangle -- in particular one with 45 degree angles for the angles that are not 90 degrees. This triple is a “1-1-\|2” right triangle, and in our case with a factor of 8 therefore giving us 8-8-8\|2, the latter value of which we just computed above.

Ok, now we know the diagonal is 8\|2. As this value is also the diameter of the circle establishing the base:

.: r = d/2 = 8\|2 / 2 = 4\|2

Now we have everything we need to get the volume of the cylinder and the volume of the cube, and subtract them!

.: Vcube = s^3 = 8^3 = 512

```
.: Vcylinder = pi x r^2 x h
             = pi x (4\|2)^2 x 8
             = pi x (16 x 2) x 8
            = 256 pi = 256 x 3.14 = 803.84
```

.: 803.84 - 512 = 291.84

Now get out the jelly! 🍞🍴

QUESTION 15.
If we solve the first part of this question logically we see that if there are 5 even integers and their average is 0, then if we look at this on a traditional number line then 0 must be the center number. And then therefore the two even integers to the right must be 2 and 4, and the two even integers to the left must be -2 and -4. And that’s our set of 5 even integers: -4, -2, 0, 2, 4

Algebraically we could solve this first part thus as an alternate approach to obtain the set of integers:

x, x + 2, x + 4, x + 6, x + 8

as 5 consecutive numbers each two apart. (There are other ways to set this up, but this to me is the clearest way.) We're told their average is 0.

$$\therefore \frac{\text{the sum of the numbers}}{5} = 0$$

$$\therefore \frac{x + (x + 2) + (x + 4) + (x + 6) + (x + 8)}{5} = 0$$

$$\frac{5x + 20}{5} = 0$$

$$5x + 20 = 0$$
$$5x = -20$$
$$x = -4$$

If again the integers were symbolically as:

x, x + 2, x + 4, x + 6, x + 8

Upon substituting x we therefore have:

−4, −2, 0, 2, 4

the same even values as we got logically above.

Now that we know the integers, let's explore the second part of this question.

Difference means subtraction.

The negative difference of the greatest and least integer:

−4 − 4 = −8

The positive difference of the greatest and least integer:

4 − −4 = 4 + 4 = 8

The sum of the negative difference and the positive difference:

−8 + 8 = 0

When you think about this logically, as we did earlier for the first part of the question, as the average is zero the integers are going to be “balanced” on both sizes of zero, therefore, the sum of the differences must also be zero since they are opposite signs and “balanced” too.

QUESTION 16.
First figure out how long it will take them to meet:
d = r x t in consideration of 10 mile lead provides: S x t = F x t + 10miles
10mph x t = 5mph x t + 10miles
5mph x t = 10miles
t = 2 hours

.: Sally ran 10mph x 2h = 20miles
Fred ran 5mph x 2h = 10m + the 10miles lead = 20miles
Sally catches up to Fred 20 miles into the race.

.: For Sally to run back 20 miles will take 2 hours again
If Fred runs for 2 hours with no lead he travels 10 miles

.: Fred still has 20 - 10 = 10 miles to go (Fred has two more hours to run too.)

.: Sally will run 20 miles on the circular track while Fred runs his last 10 miles to the finish line

1 miles = 5280 ft
10 miles = 52800 ft
52800 / 330 = 160
160 x 2 = 320 revolutions
Using π is not necessary
(And yes I could have 5280 x 20 / 330 too I just found working with multiples of 10 and multiples of 2 more handy.)

QUESTION 17.

The area of a circle is πr^2
The circumference of a circle is $2\pi r$
The area of a rectangle is $l \times w$
The perimeter of a rectangle is $l + w + l + w = 2l + 2w$

.: The area of the rectangle is $\pi r^2 \times 2\pi r = 2\pi^2 r^3$

.: The perimeter of the rectangle is $2(\pi r^2 + 2\pi r) = 2\pi r^2 + 4\pi r = 2\pi r(r + 2)$

The problem asks us to solve area of the rectangle / perimeter of the rectangle

.: $$\frac{2\pi^2 r^3}{2\pi r(r + 2)} = \frac{\pi r^2}{r + 2}$$

The problem tells us the radius is 2

.: $$\frac{\pi 2^2}{2 + 2} = \frac{4\pi}{4} = \pi$$ rounded gives an integer value of 3

Alternative/Double check:

The circumference of the circle is $2\pi r = 2\pi 2 = 4\pi$
The area of the circle is $\pi r^2 = \pi 2^2 = 4\pi$
The area of the rectangle is $4\pi \times 4\pi = 16\pi^2$
The perimeter of the rectangle is $4\pi + 4\pi + 4\pi + 4\pi = 16\pi$

$$\frac{\text{area of the rectangle}}{\text{perimeter of the rectangle}} = \frac{16\pi^2}{16\pi} = \pi$$

QUESTION 18.
A perfect square is an integer (say 9) that is the square of an integer (3^2).

What does \|144 bring us close to or exactly to? (Note \| means square root.)

You should all know 12 x 12 = 144

But what does \|1444 bring us close to or exactly to?

Well, you should also know that 40 x 40 = 1600, therefore the root sought doesn’t get as high as 40, and you know 30 x 30 = 900 which is too low even in consideration of squaring.

.: It looks like 39 wouldn’t work as it still seems too high (but it might work we just don’t know yet) so let’s try 38 as a first try, especially as 8x8=64 (using 8 as it is the ones digit from 38) and we’re looking for a 4 in the final product’s ones digit (which 64 has and will give us once the multi-digit multiplication is done). Note that we also know that it is not 39 or 37 because squares of even numbers are even:

  38 x 38 = 1444

Bingo, got it on the first try!

So we’re looking at all the perfect square integers between 12 and 38 .: from 13 to 37.

To find the range of a bunch of numbers, order them, and then subtract the highest number from the lowest number.

.: 37 - 13 = 24

But this is not our answer. This is the answer:

.: 37^2 - 13^2 = 1369 - 169 = 1200

The above used a hunt-and-peck ad hoc binary search to find the square root. A more algorithmic approach to find the square root of a 3 or 4 digit perfect square follows.

Step 1: Consider this list of perfect squares from 1 through 9:

1^2 = 1
2^2 = 4
3^2 = 9
4^2 = 16
5^2 = 25
6^2 = 36
7^2 = 49
8^2 = 64
9^2 = 81

Note: The ones column we have 1, 4, 9, 6 and then 5 and then the same list in reverse 6, 9, 4, 1

Step 2: Consider the ones column of the number in question:

.: the last 4 in 1444

Step 3: Find which perfect squares in the list above have the same ones digit

.: 2 and 8

Note: 2+8 = 10, this will always be the case in our choices, making it easier to do this

Our answer is going to end in one of these, and we will choose between them by either picking the lower one or the higher one.

Step 4: We're going to now ignore the 2 rightmost digits and focus on the 2 leftmost digits (or one leftmost digit in the case of a 3 digit number)

.: 14 for our example

Step 5: Look through the perfect square list above for a number just below our target

.: 4x4=16 is too high but 3x3=9 is the choice just just below 14

.: 3 is the first digit of our answer

Step 6: We need consider the two values from Step 3. That is, is the answer 32 or 28?

What we're going to do is multiply the first digit (3) by itself plus one (3+1 = 4)

.: 3 x 4 = 12

Step 7: Compare the value from Step 6 with the value from Step 4.

If Step 4's value is less than Step 6's value we use the lower number from Step 3.
If Step 4's value is higher than Step 6's value we use the higher number from Step 3.

.: 14 > 12 so we use 8 .: the number that was squared is 38

This process appears tedious but with practice can be a fairly seamless and fast process. However it is specific to 3 or 4 digit perfect squares.

As another example, consider 5476:

* Step 2&3: Last digit 6 means our choices are 4 or 6
* Step 4&5&6: Consider 54, 8x8=64 too high, 7x7=49 .: use 7 .: 7x8=56
* 54 < 56 .: use 4 .: 74

Consider 9801:

* Step 2&3: Last digit 1 means our choices are 1 or 9
* Step 4&5&6: Consider 98, 10x10=100 too high, 9x9=81 .: use 9 .: 9x10=90
* 98 > 90 .: use 9 .: 99

Consider 5625:

* Step 2&3: The last digit is a 5 (for this example, the complement number would have also been 5, so we're left with a 5 for sure)
* Step 4&5&6: Consider 56, 8x8=64 too high but 7x7=49 next lowest .: use 7 .: 7x8=56
* 56 = 56, besides 5 is the only choice from Step 3 .: 75

It's easy to do but it's also a specific algorithm.

QUESTION 19.

$.08\overline{3} \times .08\overline{3} = .0069\overline{4}$

Choice (A) should 6.9×10^{-3} not to mention throwing away the $\overline{4}$, so no-go.

The question did not mention rounding or truncating or estimating so (B) is out too.

(C) is a faux attempt doing $.08\overline{3} + .08\overline{3}$ but this asks for exponentiation not addition

$.08\overline{3}$ is $\frac{1}{12}$ (You should all know this.)

$$\frac{1}{12} \times \frac{1}{12} = \frac{1}{144} = 144^{-1} \qquad \text{Choice D}$$

You guys should know all your basic fractions and their decimal conversions!

If not, fear not, and convert the repeating decimal to a fraction:

$$\begin{aligned} 1000x &= 83.\overline{3} \\ -100x &= 8.\overline{3} \\ \hline 900x &= 75.0 \\ x &= 75/900 = 3/36 = 1/12 \therefore .08\overline{3} = 1/12 \end{aligned}$$

QUESTION 20.
Given: The diameter of a sphere is the diagonal of a cube.

If you take a face of the cube we see that it is $\sqrt{3} \times \sqrt{3}$. A diagonal across a face could be found by using the Pythagorean Theorem:

$$c^2 = (\sqrt{3})^2 + (\sqrt{3})^2 = 3 + 3 = 6$$
$$c = \sqrt{6}$$

You could have also calculated this considering the Pythagorean Triple $1-1-\sqrt{2}$ which would have provided you $\sqrt{3}-\sqrt{3}-\sqrt{3}\sqrt{2}$ which is $\sqrt{3}-\sqrt{3}-\sqrt{6}$.

Unfortunately this is the diagonal of the face of the cube. This is different than the diagonal of the cube itself. There are 4 diagonals to a cube. For instance, if you're facing a cube on a level flat surface one of its diagonals can be considered as a line from the top front right vertex to the bottom back left vertex. Let's call this Line D.

THE FORMULA FOR THE DIAGONAL OF A CUBE IS $s\sqrt{3}$ where s is the side length of the cube.

$\therefore$ For this cube: $s\sqrt{3} = \sqrt{3} \times \sqrt{3} = 3 \therefore 3$ is the diagonal length for this cube

$\therefore$ 3 is the diameter d of the sphere
$\therefore r = d / 2 = 3/2 = 1.5$ is the radius of the sphere

$$SA_{sphere} = 4 \pi r^2$$
$$V_{sphere} = 4/3 \pi r^3$$

$$\therefore SA_{sphere}/V_{sphere} = \frac{4 \pi r^2}{4/3 \pi r^3} = 3/r = 3/1.5 = 2$$ (Note: $3/r$ is universally true!!)

For those who did it longhand:

$$SA_{sphere} = 4 \pi r^2 = 4 \pi \, 2.25 = 9 \pi$$
$$V_{sphere} = 4/3 \pi r^3 = 4/3 \pi \, 3.375 = 4.5 \pi$$

$$\therefore 9 \pi / 4.5 \pi = 2$$

You could have also computed the cube diagonal longhand if you didn't know the formula is $s\sqrt{3}$. To do so is described below.

As it turns out the cube's diagonal can be considered the hypotenuse of a right triangle, with one leg being the side edge of the cube and the other leg being the diagonal of a face of the cube. In the example of Line D given earlier, the one leg would be the diagonal across the bottom face, and the other leg would be the front rightmost edge of the front face. We already know these two values, the former would be \|6 (which we computed) and the latter \|3 (which we were given) respectively. Therefore, we can just use the Pythagorean Theorem again:

```
  c^2 = (\|3)^2 + (\|6)^2 = 3 + 6 = 9 .: c = \|9 = 3
```

This (hypotenuse) is the same value that we computed above (for the cube diagonal), therefore the rest would be the same too.

A SHSAT problem may not always be as involved as this, however, many variant questions can be asked of you using all the different aspects of math that this problem uses.

It could also be handy to memorize the additional formulas that came up in solving this problem regarding the diagonal of a cube and also regarding the surface area of a cube to its volume in an effort to save time but as can be seen they can be computed if necessary.

You can't escape that you should know the surface area and the volume of a cube though!!!

QUESTION 21.

```
A + B = 1221   (first equation)
22A + 33B = 34188   (second equation)
```

As we did in the “Solution Mixture” and the “Drinks n Popcorn” questions, metabolize the two equations so we can throw A or B away in this case B:

```
33(A + B = 1221) = 33A + 33B = 40293
```

And subtract throwing away B:

```
  33A + 33B = 40293
- 22A + 33B = 34188
-------------------
  11A       =  6106
  A         = 555   .: B = 666
```

Double check the first equation:

```
A + B = 1221
555 + 666 = 1221
```

Double check the second equation:

```
22x555 + 33x666 = 12210 + 21978 = 34188
```

Alternative approach:

```
A = Form A        1221 - A = Form B
Total Number of Form A pages + Total Number of Form B pages = 34188
22A + 33(1221 - A) = 34188
22A + 33x1221 - 33A = 34188
22A + 40293   - 33A = 34188
-11AA + 40293       = 34188
-11A                = -6105
A = 555  The rest is as per above.
```

QUESTION 22.

If 7/11 of the problems are arbegla and there are 7 arbegla questions that would mean

7x/11 = 7 .: x = 11 total questions .: 11 – 7 = 4 would mean there are 4 others.
This rules out (A).

If 7/11 of the problems are arbegla and there are 28 arbegla questions that would mean 7x/11 = 28 .: x = 44 total questions .: 44 – 28 = 16 would mean there are 16 others.
This rules out (B).

If 7/11 of the problems are arbegla and there are 22 arbegla questions that would mean 7x/11 = 22 .: x = 34.57 total questions .: 34.57 – 22 = 12.57 would mean there are 12.57 others.
This rules out (C).

If 7/11 of the problems are arbegla and there are 35 arbegla questions that would mean 7x/11 = 35 .: x = 55 total questions .: 55 – 35 = 20 would mean there are 20 others.
This is our answer (D).

QUESTION 23.
It’s easy to think this is the average of 4 + 6 or 10/2 = 5. But that would be wrong.

distance = rate x time ala d = r x t .: r = d/t .: t = d/r

de Blasio’s total distance is the distance from the coffee shop to the point where he gets hungry in Prospect Park plus the distance to return via the same route.
.: distance going + distance return = distance x 2

Similarly, he has two times, the time it took him to go from the coffee shop into Prospect Park at 4mph:

$$\text{time going} = \frac{\text{distance going}}{\text{rate going}} = \frac{\text{distance}}{\text{4mpg}}$$

And the time it took him to go from Prospect Park back to the coffee shop via the same route:

$$\text{time returning} = \frac{\text{distance returning}}{\text{rate returning}} = \frac{\text{distance}}{\text{6mph}}$$

$$.:\ \text{time overall} = \frac{\text{distance}}{\text{4mph}} + \frac{\text{distance}}{\text{6mph}} \quad \text{so we'll use this below}$$

This means his overall rate for the while trip is:

$$\text{rate overall} = \frac{\text{distance x 2}}{\dfrac{\text{distance}}{\text{4mph}} + \dfrac{\text{distance}}{\text{6mph}}} = \frac{\text{distance x 2}}{\dfrac{\text{distance x 6}}{\text{24mph}} + \dfrac{\text{distance x 4}}{\text{24mph}}} = \frac{\text{distance x 2}}{\dfrac{\text{distance x 10}}{\text{24mph}}} =$$

$$= \frac{\text{distance x 2 x 24mph}}{\text{distance x 10}} = \frac{\text{48mph}}{10} = \text{4.8mph} \qquad \text{Choice A}$$

QUESTION 24.
We all know that:

distance = rate x time ala d = r x t .: r = d/t .: t = d/r

.: de Blasio’s rate = 12 / 2 = 6mph

.: Carranza’s distance = 12 / 3 = 4 miles
Carranza’s time = 2 x 2 = 4 hours
Carranza’s rate = 4 / 4 = 1mph

.: Carranza’s rate is 6mph / 1mph = 6 times slower

.: de Blasio will return 6 times slower Choice H

QUESTION 25.
Following the gripping unfolding story, there is:
3 chocolate
5 raspberry
4 lemon
4 glazed
————
16 total

He takes one of each leaving:
2 chocolate
4 raspberry
3 lemon
3 glazed
————
12 total

2 more chocolate are added:
4 chocolate
4 raspberry
3 lemon
3 glazed
————
14 total

And then 2 sleigh and reindeer croissants for a total of 16

So picking a chocolate at this point is 4 out of 16 = 1/4 or 0.25

QUESTION 26.
The statement gives us the following sub-statements:

A = 99B
A = C - 998
A = D/999

I don't like the constants here but let me try to see if using B = 10 as a test candidate will turn out clean:

A = 99B = 99 x 10 = 990

A = C − 998
990 = C − 998
990 + 998 = C
1988 = C

A = D/999
990 = D/999
990 x 999 = D
989010 = D

B is the smallest value and will remain so even with other test numbers given our constraint "> 0"

We could have also approached the problem in another way. If you got rid of the fraction we'd have:

999(A = 99B = C − 998 = D/999)
999A = 999x99B = 999C − (999x998) = D
999A = 98901B = 999C − 997002 = D

It should be clear that:
D > A and D > B as D has no coefficient
A > B as its coefficient is smaller than Bs coefficient .: A is larger than B

That leaves B pitted against C with 98901B = 999C − 997002

But if B is say 1 then C must be at around 1000 to surpass the negativity of 997002 but that interim expression value will still be off by many magnitudes less than it needs to be .: Choice B

QUESTION 27.
If Chuck already chucked 3 pieces then he has 120 pieces left.

If 1/6 is elm then that's 1/6 of 120 = 20 pieces
If 1/5 is oak then that's 1/5 of 120 = 24 pieces
If 1/4 is maple then that's 1/4 of 120 = 30 pieces

20 + 24 + 30 = 74 pieces .: 46 pieces are other types of wood

To ensure a piece of oak he must chuck at least all of the elm, maple, and other pieces

.: 20 + 30 + 46 = 96 pieces

The next piece would guarantee an oak piece was chucked, so 97 pieces   Choice C

QUESTION 28.

$$\text{Average} = \frac{\text{sum of the numbers}}{\text{number of numbers}}$$

$$500 = \frac{\text{y1sum}}{600}$$

.: y1sum = 500 x 600 = 300000

$$600 = \frac{y2sum}{500}$$

.: y2sum = 600 x 500 = 300000

$$Ay1y2 = \frac{y1sum + y2sum}{numy1 + numy2}$$

$$Ay1y2 = \frac{300000 + 300000}{500 + 600} = \frac{600000}{1100} = 545.\overline{45}$$ Choice B

QUESTION 29.
LEFT is similar to RIGH

```
                          9ft
     6ft          R+----------------+I
L+---------+E      |                |
 |         |       |                | x ft
 |         | 4ft   |                |
T+---------+F     H+----------------+G
```

Since similar shapes set up a proportion between the rectangles' respective sides
.: 6:9 as 4:x (can reduce 6:9 to 2:3 if you want, I just left it as is)
.: Doing cross product yields 36 = 6x (12 = 2x if you reduced)
.: x = 6
.: Width of RIGH is 6 ft
.: Aleft= $6 \times 4 = 24\ ft^2$.: Arigh = $9 \times 6 = 54\ ft^2$

.: 24:54 = 4:9 = 4/9 = $.\overline{4}$ Choice B

QUESTION 30.
If we want an estimate we can round up and have 200 x 2000 = 400000. But the answer choices given are too close together so we have to go a step further.

The most obvious knee-jerk approach is to just do the multi-digit multiplication, as we can round up or down once we get the product:

```
    199.9
   x 1999
   ------
    17991
   17991
  17991
+ 1999
---------
  3996001 accommodating the decimal point yields 399600.1 rounded is    Choice E
```

That computation is error prone, tedious, and wastes time.

We can also look at this problem logically. If 1999 is one less than 2000 that's 1/2000 less than the product therefore 400000/2000 = 200 less overall. Similarly if 199.9 is .1 less than 200 that's .1/200 less than the product. Well .1/200 is also the same as 1/2000 which we just computed therefore that's 200 less overall too. So we have 400000 - 400 = 399600.

Note that in the computations in the previous paragraph we approached as 1999x200 and 199.9x2000 so we took too much away. We need to add back 1/2000 x 1/2000 = 1/4000000 = 0.00000025 and that multiplied by 400000 is .1 The answer choices don't care about a value that small.

Lastly, it seems the logical solution got somewhat carried away itself. Turns out that it is actually accommodated mathematically/symbolically via the distributive property/FOIL.

This can apply to the problem at hand, that is, we see that

```
  199.9 is the same as 200 - .1
  1999 is the same as 2000 - 1

.: (200 - .1)(2000 - 1) = 200x2000 + 200x(-1) + (-.1)x2000) + (-.1)(-1)
                       = 400000 -200 -200 + .1
                        = 399600.1 rounded is       Choice E
```

Use the math you know to your advantage! Once you see the FOIL you can often solve this problem in your head.

QUESTION 31.
After substitutions the question “replaces to”:
A test has 20 questions. Answering a question right is worth 5 points. Answering a question wrong is worth 0 points. Not answering a question is worth 2 points. If a student answered 10 questions and got 5 of them correct, what was the student's score?

There were 20 questions and 10 answered .: 10 not answered

.: 10 unanswered x 2 = 20 points

Also, of the 10 answered got 5 correct .: got 5 wrong

.: 5 correct x 5 + 5 incorrect x 0 = 25

.: total = 20 + 25 = 45

To instead solve this graphically, we can consider a question is either correct, unanswered, or wrong along with its values:

```
    qv  (question values)
  / | \
 ct un wr
 5  2  0
```

Graphically as well, a question is either answered or unanswered, and if it’s answered it’s either correct or wrong:

```
     qt  (questions taken)
    /  \
  ans unans
 /  \
ct  wr
```

So let’s add values to this second hierarchy with response counts per those categories:

```
    20qt
    /  \
10ans 10unans
```

```
 /  \
5ct  5wr
```

Now that we understand those distributions let's consider the score values:

```
    20q
  / | \
 ct un wr
 5  2  0  (value per question type)
 5 10  5  (number within each type)
5x5 + 2x10 + 0x5 = 25+20 = 45 points
```

QUESTION 32.
In this kind of question, it's requiring you to take the time to evaluate every answer choice. It's potentially a time waster for sure. So let's go through each in turn:

Which of the following expressions is equal to an odd number?

A) 99 x 1234

This requires a multi-digit multiplication. Boo.
It evaluates to 122166.

B) 6543 – 4567

This requires subtraction with borrowing. Boo-hiss.
It evaluates to 1976.

C) 66^2

This again requires a multi-digit multiplication. Grrrr.
It evaluates to 4356.

D) 9991 / 99

A multi-digit division. Oh man.
This evaluates to 100.919192

E) 123 x 4567

Here we go again.
This evaluates to 561741

So the answer is E. However, if you notice, the only thing that really matters is the last digit. In other words, we really do not need to do all these computations. It's bad enough the question is asking us to interrogate each expression but to solve them in full is a sure time killer.

So don't. Instead, let's see how we could optimize things. Taking each in turn again. Which of the following expressions is equal to an odd number?

A) 99 x 1234

Since the ones digits involved is 9 and 4, we multiply them: 9x4=36 This means the ones column of the product must be a 6. This is even. Move on.

Note that with some products we could use the 99 to our advantage. That is, make it 100 and instead do 100 x 1234 and then subtract one 1234 from the product. But in this example that's a solution worse than the problem as 123400 – 1234 tends to be an error prone and annoying subtraction. But keep this idea in your back pocket.

B) 6543 – 4567

Once you see there will be a carry just do it (the 4 in the 10s column becomes 3 and the 3 in the ones column becomes 13 upon which we subtract 7) and you're done. That yields a 6 in the ones column. Move on, move on down.

C) 66^2

As with (A) all we care about is the 6s in the ones columns so we have 6x6=36 and the ones column of the product, another 6, is even. Move way down.

D) 9991 / 99

This requires you to do the actual division in part. 99 divides into 99 once. Then you immediately see that 99 doesn't divide into 91. So this can't be an integer and therefore can't be an odd number.

It might be tempting to look at this problem backwards using the logic for (A) and (C), hence figuring 99 x something-ending-in-a-9 will give us something ending up with 9x9=81, and oh yeah we got the 1! But math don't play like that. Be careful!

E) 123 x 4567

Finally as with (A) and (C) 3x7=21 and the ones digit is a 1 and that's odd! Don't move anywhere, you got it!

With these insights you can solve this problem in a few seconds and more cleanly.

Remember your math mastery is just as important as your math knowledge as then you can use it in many contexts, which is exactly in part what math is for and how it should be used! Furthermore, it's one aspect of what this test is testing!

QUESTION 33.
75% of 36 cookies means 36 x .75 = 27 cookies available
Instead of doing multi-digit multiplication quickly note that this is

```
36   3
-- x - cancelling the 4 into 36 giving 9 x 3 = 27
 1   4
```

80% of 30 kids means 30 x .8 = 24 kids in attendance

One third of those in attendance wanted an Oreo .: 24 / 3 = 8 wanted an Oreo

8 put out on plate - 3 left = 5 taken and eaten

If there were initially 27 Oreos and 8 were taken out and 3 put back there is now 22. You can also look at it this way:

27 - 5 = 22 Oreos left

As the package could hold 36 .: 36 - 22 = 14 empty spaces

QUESTION 34.
To find out how often they meet we could create a list and just see when they match. That can be tedious. Instead let's get the LCM. The prime factors of 21 and 18 are:

```
 21          18
 /\          /\
```

```
3  7        2  9
              /\
             3  3
```

21: 3, 7
18: 2, 3, 3
.: LCM = $2 \times 3 \times 3 \times 7 = 126$

We could have also computed the prime list with a table:

```
2|21  18
3|21   9
3| 7   3
7| 7   1
 | 1   1
```

This means the busses meet ands leave together every 126 minutes

As 1 day = $24 \times 60 = 1440$ min .: $1440 / 126 = 11.42$

That's 11 times for sure, plus midnight .: 11 + midnight = 12

QUESTION 35.
We can "easily" solve this algebraically. Four multiples of 7 can be added thus:

$7x + 7(x + 1) + 7(x + 2) + 7(x + 3) = 210$

```
.: expanding:
7x + 7x + 7 + 7x + 14 + 7x + 21 = 210
28x + 42 = 210
     -42   -42
28x = 168
/28   /28
x = 6
```

.: $7x = 42$

.: Our numbers are: 42, 49, 56, 63
$42 + 49 + 56 + 63 = 210$ checks out

.: The third multiple is: 56

An alternative solution to this part of the problem follows. As 210 is the sum of the 4 multiples, let's just get their average:

  210 / 4 = ? Well / 2 = 105 and /2 again = 52.5

So we'd have:

  first, second, third, fourth

52.5 is smack between second and third.

.: a multiple of 7 immediately higher than 52.5 must be the answer.
.: Third is 56

You could also brute force it by maybe starting with a guess of 50, bringing that up or down to a multiple of 7, and then figure which should be the first (or last) in order to obtain the multiples of 7 sum sought.

We still need to obtain the sum of the factors of the third multiple.

The factors of 56 are: 1, 2, 4, 7, 8, 14, 28, 56 (it's easy to skip some!)
Their sum is 120

QUESTION 36.
Let d be the number of days in the study plan. Spending one-third of their time on ELA reading comprehension, one-sixth of their time on ELA grammar, and 12 days of their time on math multiple choice can be represented as:

$$\frac{d}{3} - \frac{d}{6} - 12$$

Therefore if d is the total days, the remaining days thus far are:

$$d - \frac{d}{3} - \frac{d}{6} - 12$$

We can simplify this, so getting a common denominator gives:

$$\frac{6d}{6} - \frac{2d}{6} - \frac{d}{6} - 12 = \frac{d}{2} - 12 = r \text{ (r is the remaining days)}$$

If the student then planned to spend one-sixth of their remaining time on math grid-ins some solvers of this problem would just do r/6. But we're subtracting from the remaining not just getting 1/6 of the remaining, so what we're looking for is 5/6 of r not 1/6 of r. So that gives us:

$$\frac{5}{6}\left(\frac{d}{2} - 12\right) = 25$$

Multiplying both sides by 6/5 gives:

$$\frac{d}{2} - 12 = 30$$

d/2 = 42 .: d = 84 days total

Double checking:
1/3 of 84 = 28
1/6 of 84 = 14
12 days   = 12
            --
            54 days

84 - 54 = 30 remaining days
1/6 of 30 = 5
30 - 5 = 25 days (same as 5/6 of 30)

QUESTION 37.
As you’re trying to get a fair amount, you’re going to consider this linearly, therefore we’re going to be considering a line ala:

$$y = mx + b$$

in our case:

$$p = mq + b$$

where p is the price and q is the quantity.

We can figure out m, our slope, by looking at Pete's offering as pairs: 30@$10 and 45@$8 as (30, 10) and (45, 8) respectively. Therefore, our rise/run is change in price/change in quantity:

$$\frac{8 - 10}{45 - 30} = \frac{-2}{15}$$

Therefore we have

$$p = \frac{-2}{15} q + b$$

We can plug in one of the coordinates from above to calculate b:

$$10 = \frac{-2}{15} (30) + b$$

$$10 = -4 + b$$
$$14 = b$$

That gives us:

$$p = \frac{-2}{15} q + 14$$

Checking it with Pete's other offer we get:

$$8 = \frac{-2}{15} (45) + 14$$

$$8 = -6 + 14$$
$$8 = 8$$

Now using our $11 offer:

$$11 = \frac{-2}{15} q + 14$$

$$-3 = -\frac{2}{15} q$$

q = 45 / 2 = 22.5 extra large rice balls to request of Pete

We can also solve this problem logically. If $8 gives 45 and $10 gives 30 that would mean $9 would give 45+30/2 = 75/2 = 37.5 pieces (rice balls). Note that 37.5 is 7.5 away from 30 and also 7.5 away from 45. That means 7.5 per $1, therefore to go to $11 is 7.5 pieces less than the 10 price count .: 30 - 7.5 is 22.5

QUESTION 38.

Let r represent those opting for remote learning and i represent those wanting in-person learning. We know their total:

A) $r + i = 36$

We also know their relationship:

B) $i = r/3$

Replacing i in (A) with (B) we get:

$r + r/3 = 36$
$4r/3 = 36$
$r = 27$.: $i = 9$

Check: 9 (in-person) is 1/3 of 27 (remote)

We can also solve this by considering that given the relationship is 1/3, or more clearly 1:3, that there are 4 parts. The total divided by 4 will provide us each part: 36/4 = 9. The question does not ask for this value, but the other 3 parts is 3x9 and that is 27. 27 + 9 = 36

QUESTION 39.
Since the buy price is the same for all stock purchases, we only need to compare the sell prices and see which is the smallest value.

One way to solve this is to do each division and compare the decimal values. I'll only write out as many decimal places to make each value distinguished from the others in cases where the decimal digits might be many digits:

A 9/30 = 3/10 = .3

B 5/12 = $.41\overline{6}$

C .36

D 9/24 = 3/8 = .375

E 15/48 = 5/16 = .3125

The smallest value is for A.

Another way to solve this is by comparing each fraction pair in turn against each other using the cross product. Whichever is smallest, compare that to the next fraction until there are no more choices left. That is:

9/30 ? 5/12 .: 108 vs 150 .: 9/30 is the smallest
9/30 ? .36 --> 9/30 ? 36/100 .: 900 vs 1080 .: 9/30 is the smallest still
9/30 ? 9/24 .: 216 ? 270 .: 9/30 is the smallest
9/30 ? 15/48 .: 432 vs 450 .: 9/30 is the smallest

Although I started comparing against A, it would be the same process no matter which you picked first, although with a different set of numbers one of them may have leaped out as a possible better choice to start with.

Also, in these comparisons, you could have worked with simpler numbers when doing the cross products by simplifying some of these fractions but on the flip side that may have added extra time. Maybe.

Yet another way to solve this was to note that each was close to being 1/3. This strategy often turns out to be the quickest approach. In fact each fraction was 1/x away from 1/3 where x is the denominator being used by the choice. In other words:

For A 9/30 it is 1/30 less than 10/30
For B 5/12 it is 1/12 more than 4/12
For D 9/24 it is 1/24 more than 8/24
For E 15/48 it is 1/48 less than 16/48

B and D are over 1/3 hence out of the running since there are choices less than 1/3. A and E are under 1/3 but 1/48 is closest to 1/3 than 1/30 as it has a larger denominator. This means 9/30 is further away and therefore the smallest value of all the fractions.

This leaves comparing 9/30 to .36 as our final comparison. As .36 is 36/100 that comes into play because it is about 2 2/3 away from 33 1/3 divided by 100 which means, as with the E comparison, that C is closer to 1/3 than A, therefore A is the smallest of all the fractions and the .36 decimal. Therefore, A is the largest loss.

My stock loss on A is .03 but that is not what the question asks.

It’s not so much this question is hard, it’s that it’s annoying.

QUESTION 40.
25% of 312 is 78

(A) twos + fours = 24
.: fours = 24 − twos

(B) 2 x twos + 4 x fours = 78
Substitute .: 2 x twos + 4(24 − twos) = 78
2 x twos + 96 − 4 x twos = 78
−2 x twos = −18
.: twos = 9 .: fours = 15

Check:
9 + 15 = 24
2(9) + 4(15) = 18 + 60 = 78

We could have also solved this by normalizing (A) against (B) and cancelling out either twos or fours, so let’s get rid of fours by multiplying (A) by 4:

4(twos + fours = 24)
(A’) 4 x twos + 4 x fours = 96

Now let's subtract (B) from (A’):

```
  4 x twos + 4 x fours = 96
- 2 x twos + 4 x fours = 78
---------------------------
  2 x twos             = 18
```

.: twos = 9, as we got earlier

QUESTION 41.
Note: \| represents square root, same as \2| does, .: \3| represents cube root

\|0.16 is 0.4 However there is a negative sign so we’re looking at −0.4

We need to do the reverse operations here. The reverse of the cube root is cubing and the reverse of increasing by 1 is decreasing by 1.

.: We get: r^3 – 1
.: (–0.4)^3 – 1 = –0.064 – 1 = –1.064

Checking:
if n is –1.064 .: n + 1 = –0.064
\3|–0.064 = –0.4
0.4^2 = 0.16 .: square rooting both sides .4 = \|0.16 .: –.4 = –\|.16

There will be a tendency to solve this one in your head. But it's easy to mess up the square and cube root and to mess up the squaring and cubing and to mess up the decimal place and to mess up adding or subtracting 1. Attention to detail and numeracy is important when problem solving.

QUESTION 42.
The area of a trapezoid, whether isosceles or not, is the average of its bases times its height:

$$A = \frac{(b1 + b2)}{2} \times h$$

If the bases double and the height halves that give us:

$$A = \frac{2b1 + 2b2}{2} \times \frac{h}{2} = \frac{2(b1 + b2)}{2} \times \frac{h}{2}$$

That simplifies to:

$$A = \frac{(b1 + b2)}{2} \times h$$

which is the original equation. .: The area stays the same Choice E

By the way if there was a thought that the area halved, well, Choice B and Choice D both represent that, therefore those wouldn't be the case.

We could have also brute forced some sample numbers say a top base of 2 a bottom base of 4 and a height of 10. That would yield an area of 3 x 10 = 30 units.

If we doubled the bases we'd get 4 and 8 and if we halved the height we'd get 5; the area in that case would be 6 x 5 = 30 units as well.

If you didn't know the direct formula to compute the area of a trapezoid, you could have also broken down the problem into three parts: a rectangle, with two right triangles adjacent to it. The left and right triangles are congruent as this was an isosceles trapezoid, but that would turn out neutral information depending upon how you solved this. Using "the normal formulas" we'd get:

Ainitialrectangle = l x w = b1 x h
Ainitiallefttriangle = bh/2 = ((b2 – b1)/2) x h / 2 = h(b2 – b1)/4
Ainitialrighttriangle = bh/2 = ((b2 – b1)/2) x h / 2 = h(b2 – b1)/4
Ainitiallefttriangle + Ainitialrighttriangle = h(b2 – b1)/2
Aall = b1 x h + h(b2 – b1)/2

Arevisedrectangle = l x w .: for us: 2b1 x h/2 = b1 x h
Arevisedlefttriangle = bh/2 .: for us: (2(b2 – b1)/2) x (h/2) / 2 = (b2 – b1) x h / 4
Arevisedrighttriangle = bh/2 .: for us: (2(b2 – b1)/2) x (h/2) / 2 = (b2 – b1) x h / 4
Arevisedlefttriangle + Arevisedrighttriangle = h(b2 – b1)/2
Aall = b1 x h + h(b2 – b1)/2

Or using test numbers with this approach:

Ainitialrectangle = 2 x 10 = 20
Ainitiallefttriabgle = (4−2)/2 = 1 x h = 1 x 10 = 10 / 2 = 5
Ainitialrighttriangle = 5
Aall = 20 + 5 + 5 = 30

Arevisedrectangle = 2x2 x 10/2 = 4 x 5 = 20
Arevisedlefttriangle = 2x4 − 2x2 = 8 − 4 = 4 /2 = 2 x 10/2 = 10 / 2 = 5
Arevisedrighttriangle = 5
Aall = 20 + 5 + 5 = 30

QUESTION 43.
A goal is to interconnect the parts relationship as x : y : z

We're told: x : y is 2 : 3

Also:
If z to y is 1/3 : 0.5 .: also 1 : 1.5 .: also 2 : 3
If z to y is 2 : 3 .: y to z is 3 : 2

.: combining x : y with y : z we get: x : y : z and that is 2 : 3 : 2

2 : 3 : 2 indicates 7 parts

.: total / parts = 4.9 / 7 = 0.7

0.7 x 3 = 2.1 y's

checking:
0.7 x 2 = 1.4 x's and 1.4 z's

Add the parts:
1.4 + 2.1 + 1.4 = 4.9

Further checking:
x : y is
1.4 : 2.1 dividing each by 0.7 is 2 : 3

z : y is
1.4 : 2.1 dividing each by 4.2 is 1/3 : 0.5

QUESTION 44.
Every time they said it, there is the two weeks they claim, plus one more week, therefore that's three weeks per utterance.

Since today is the fifth time, they've done it 4 times previously. That's 12 weeks. As the fifth time was today we don't need to add any further two weeks or additional week. Choice I.

QUESTION 45.
We can apply the Pythagorean Theorem c^2 = a^2 + b^2:

$(x - 2)^2 + (x - 4)^2 = x^2$

$(x - 2)^2 = x^2 - 4x + 4$
$(x - 4)^2 = x^2 - 8x + 16$

.: $x^2 - 4x + 4 + x^2 - 8x + 16 = x^2$
$2x^2 -12x + 20 = x^2$
$x^2 -12x + 20 = 0$
$(x - 2)(x - 10)$.: $x = 2$, $x = 10$
It’s not possible for x to be 2 as that would yield leg lengths less than zero
.: $x = 10$

Check:
Replacing x with 10 gives us a 6–8–10 right triangle, which is the Pythagorean triple 3–4–5 x 2. You should have Pythagorean triples in the back of your mind when doing questions such as this one. No guarantee it will be the case, but a good likelihood it will be.

QUESTION 46.
We just have to do the operations in reverse order to solve this:

\|729 = ?

You should know 30 x 30 is 900 (too high) and 25 x 25 = 625 (too low).

To get the 9 in the ones column of 729 we must be multiplying something ending in a 3 hence 3 x 3 or ending in a 7 hence 7 x 7. Since 25 was too low then 23 is too low too and therefore won’t work either, so there’s a pretty good possibility it’s 27 and it is:

\|729 = 27

27 is small enough that “we got this” to figure out i^i:
$3^3 = 27$

.: $i = 3$

This problem maps into 3^6 and other possibilities, but they don’t help us much to reverse 729 back to 3 without going beyond the scope of the problem if you will.

QUESTION 47.
The aquarium and water being 20 pounds together can be expressed as:

$a + w = 20$

The aquarium with half the water can be expressed as:

$a + w/2 = 15$

Through various techniques we can see that w/2 must be 5.
.: $w = 10$

Both the aquarium and the water needed for complete volume each weighs 10 pounds respectively.

The aquarium weighs the same whether it is upright or upside down, so that doesn’t matter.

.: The aquarium with a 5 pound rock atop it would be 10 + 5 = 15 pounds

QUESTION 48.
This is simply the distributive property:

$a (b + c) = ab + ac$

in reverse, that is:

$ab + ac = a (b + c)$

In our case we have 33x33 + 33x34, that’s 33 (33 + 34), and that’s 33(67)

Now all we have to do is match

$67 = 66 - \Delta$

.: $\Delta = -1$

This should be a no hesitation 20 second problem! Next!

QUESTION 49.
At first glance this looks weird and perhaps not even possible.

Don’t think it’s 0 because division by zero is not possible as it’s undefined behavior which would make the left fraction nonsense as well as the fraction that's the denominator of the greater right fraction.

We can “normalize” the greater right fraction by noticing that it's

1 divided by $\frac{48}{x}$

And division by a fraction is that same as multiplication by its reciprocal. If we do that, we get:

$1 \times \frac{x}{48}$

.: our problem becomes

$$\frac{3}{x} = \frac{x}{48}$$

This still looks weird and perhaps not even possible, and x as 0 still is inappropriate.

If we now do the cross product we get:

$x^2 = 3 \times 48 = 144$
If we take the square root of both sides .: $x = 12$

Note that x is also -12. It asks for the smallest value and $-12 < 12$.

This should be a 20 second problem. There are other variations to the steps in solving this but nothing earth-shattering in the variations. Feel free to play with this.

Perhaps the test will most likely not ask you this question so straightforward as I have here. However, it could be something you need to come up with in the middle of solving a word problem and it shouldn't throw you off.

QUESTION 50.
One approach to this question is to just do the variable replacements. That gives:

$a - (a/2 - b/2) = (a - x) - b/2$

Solving for x gives:

```
a - a/2 + b/2 = a - x - b/2
    a/2 + b/2 = a - x - b/2
-a             -a
   -a/2 + b/2 =    -x - b/2
         +b/2         +b/2
   -a/2 + b   =    -x
    a/2 - b   =     x
```

That looks like we boxed ourselves in with nonsense but then you realize you can replace b again, so you get:

```
    a/2 - a/2 = x
            0 = x
```

That's probably going to be error-prone on test day.

Another observation is that if b is a/2 and c is b/2 then c is a/4. Does that help us? Let's see:

$a - (a/2 - a/4) = (a - x) - a/4$

Well, looks like we have to play some games with common denominators. Let's do so:

```
a - (2a/4 - a/4) = 3a/4 - x
a - a/4          = 3a/4 - x
3a/4             = 3a/4 - x
-3a/4              -3a/4
0 = -x .: x = 0
```

This approach gives us some insight into the problem that may have leaped out from the get-go: That b - c must be c if b = a/2 and c = b/2(or a/4).

And if b - c must be c, then x must be zero. Ya? Consider the original equation:

$a - (b - c) = (a - x) - c$

Substituting:

```
a - c = a - x - c
a - c = a - c - x
0     =       - x
```

QUESTION 51.
If I placed a bet of $25 thinking the profit would be the bet squared, that would be

25 x 25 = 625 (know your perfect squares fluently!).

However, if the profit is the square root of the bet, that would be \|25 = 5.

The average of 625 and 5 is (625 + 5)/2 = 630/2 = 315.

The range of a set of numbers is the maximum value - minimum value. In our case the set is { 625, 5 } and thereof the range is 625 - 5 = 620.

The difference of two numbers means subtraction. The positive difference is the value a - b and b - a being taken as the same in effect |a - b| is the same as |b - a|. This can be done simply by subtracting the smaller number from the larger number, in our case 620 - 315 = 305

305 is the answer.

QUESTION 52.
As the floor size is 135m x 79m and the overhang is 24cm, then the size of the initial tarp is

(135m + 24cm + 24cm) x (79m + 24cm + 24cm) = 135.48m x 79.48m = 13548cm x 7948cm.

Adding an additional 24cm to each edge around the tarp would yield a modified new tarp size

(13548 + 24 + 24) x (7948 + 24 + 24) = 13596cm x 7996cm

So that perimeter would be 2 x 13596cm added to 2 x 7996cm = 27192 + 15992 = 43184cm

But the problem doesn't ask for the new perimeter after the additional tarp but what the lengths of the additional tarp "border" itself would be.

To do this we need two lengthwise pieces and two width-wide pieces of this border each with their own width of 24cm. Because the border pieces will not overlap either the two length-wise pieces will each have 24+24=48cm added to them but the width-wise pieces won't, or the two width-wise pieces will each have 48cm added to them but the length-wise pieces won't.

This gives us either:

2(13548 + 24 + 24) + 2(7948) = 27192 + 15896 = 43088cm
or
2(13548)   + 2(7948 + 24 + 24) = 27096 + 15992 = 43088cm

QUESTION 53.
The total of the numbers in set L is 256

There are 11 numbers in set L

.: average of L is 256 / 11 = $23.\overline{27}$

That's unhelpful though. However, if we divide the total by the target average we get:

256 / 22 we get $11.\overline{63}$

Using the whole number part of that means 22 x 11 = 242 of the 256 is accounted for.

If we subtract that from 256 we get 256 - 242 = 14

14 is not one of the numbers. But if we add 22 to it we get 14 + 22 = 36 and 36 is the number that throws off the sought average of 22. We added 22 because when we divided 256 it turns out that 22 units of the 36 got included in that as 36/22 ~= 1.63.. in other words one 22 plus about 63/100 of a 22 (and about 63/100 of a 22 is the 14).

Another way to approach this was to note that we could make pairs within the set that add up to 44. This is significant because 44/2 is 22. So once we pair those up, whatever is unpaired and/or not 44 might be the candidate we're looking for.

So keeping the set ordered for convenience, the pairs of 44 we get are 7/37, 11/33, 13/31, 16/28, and 19/25. The only one outstanding number left is 36.

Another way it to remove each number in turn and calculate the average. This is tedious, error-prone, and slow.

Saving the best for last: If one number doesn't belong, and there are 11 numbers in set L, then only 10 numbers belong. If the average of those 10 number is 22, that means their sum must be 220. But the total of the 11 numbers was 256, therefore 256 - 220 = 36. Ba-ba-ba-booyah!

This tends to be more a number sense problem and logical reasoning problem in addition to math notions such as averages and sets.

QUESTION 54.
Given speed S1 and speed S2, the average speed OVER THE SAME TIME is:

$$am = \frac{S1 + S2}{2} \quad \text{Arithmetic mean computation}$$

So if I drove an hour at 10mph and an hour at 15mph, then my average speed with be 12.5mph.

However, when we're looking for the average speed OVER THE SAME DISTANCE that does not work. Instead we need to use the reciprocal of the normal average of the reciprocals of the speeds. That is to say in math:

$$\frac{2}{\frac{1}{S1} + \frac{1}{S2}} \quad \text{Harmonic mean computation}$$

This is often termed a few things, one being the "harmonic mean."

Therefore, in our example that would be:

$$\frac{2}{\frac{1}{10} + \frac{1}{15}} = \frac{2}{\frac{15}{150} + \frac{10}{150}} = \frac{2}{\frac{25}{150}} = \frac{2 \times 150}{25} = 12$$

In addition to the classic formula above for the harmonic mean there is also a variation formula:

$$\frac{2 \times S1 \times S2}{S1 + S2}$$

Using our numbers we get:

$$\frac{2 \times 10 \times 15}{10 + 15} = \frac{30 \times 10}{25} = \frac{300}{25} = 12$$

Pick which you prefer, or use both to double check your computations.

Again, of importance here with these formulas is that the distance remains constant and not the time. This means we are not computing an arithmetic mean (what you know and love thus far) but a harmonic one. In short, instead of weighting for time we're weighting for distance.

Where are all these obtuse formulas coming from? Good ol' "d = r x t"!

If d = r x t then we also have:

$$r = \frac{d}{t} \quad \text{and} \quad t = \frac{d}{r}$$

Consider:

The time to go is:

$$Tg = \frac{d}{10}$$

The time to return is:

$$Tr = \frac{d}{15}$$

.: the total time, Tt, is Tg + Tr:

$$Tt = \frac{d}{10} + \frac{d}{15} = d\left(\frac{1}{10} + \frac{1}{15}\right)$$

Our average speed must consider the distance going and the distance returning, in other words the total distance is 2d since each respective distance is the same length.

Since r = d / t we're looking for the total distance over the total time. We know both of those, so let's just do it:

$$\text{Average total speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2d}{Tt} = \frac{2d}{d\left(\frac{1}{10} + \frac{1}{15}\right)} = \frac{2}{\frac{1}{10} + \frac{1}{15}}$$

This is none other than the particular harmonic equation we pulled out of a hat and used earlier!

If it’s too conceptual approaching this as strictly as the above (no doubt the formula is looking weird), you could always plug in real numbers. Since the two speeds we’re looking at are 10 and 15, then just pick a number that is a multiple of them. In particular it is often useful that the chosen distance just be their LCM. In this case the LCM is 30. This will represent the distance involved. We don’t know the real distance but it does not matter; all that matters is that we maintain the same distance going and returning!

As t = d / r this means Tg = 30 / 10 = 3 and Tr = 30 / 15 = 2

.: Tt = Tg + Tr = 3 + 2
.: Total Distance Dt = 30 + 30 = 60

As r = d/t .: 60 / 5 = 12

QUESTION 55.
Let's consider the following: If something takes 30 minutes, then doing is three times as fast means it takes 10 minutes to be done, doing it twice as fast means it takes 15 minutes, and doing it half as fast means it take 60 minutes. Twice as fast (200% as fast) is not the same as half as fast (50% as fast).

Because revising and editing (re) is half as fast as reading comprehension (rc) this means it is slower, re takes twice as long as rc. Similarly if re is 3 times as fast as the math (m) then re takes 1/3 as long as m.

.:
The ratio of re to rc is 2 : 1

The ratio of re to m is 1 : 3

Visually this gives us:

```
re : rc
2  : 1
re      : m
1       : 3
```

To metabolize these different units we can make a conversion for re : m by x 2:

```
re : rc
2  : 1
re     : m
2      : 6
```

Together that would give us:

```
re : rc : m
2  :  1 : 6
```

Which is comprised of 9 parts altogether.

Next: 3 hours = 60 x 3 = 180 minutes

180 minutes – 45 minutes to double check = 135 minutes

135 / 9 = 15 minutes per part

.:
re is 15 x 2 = 30 min This is the answer
rc is 15 x 1 = 15 min
m is 15 x 6 = 90 min
(We didn't need to compute the other two except to double check)

Double checking: 30 + 15 + 90 + 45 = 180 min / 60 = 3 hours

QUESTION 56.
This example is based on problem 68 of the 2020–2021 SHSAT Handbook. I normally do

problems from scratch but figured it was worth pointing out this problem, and of course add a spin to it. Phrasing and attention to detail matter.

If 1/4 of the water (note this does not say 1/4 cup of water, but 1/4 of the total water) in Jar 1 is transferred that is

1/2 x 1/4 = 1/8

So Jar 1 now has 1/2 - 1/8 = 4/8 - 1/8 = 3/8 of water left

And Jar 2 now has 1/2 + 1/8 = 4/8 + 1/8 = 5/8

If both their current volume is increased by 4-fold that means

Jar 1 = 4 x 3/8 = 12/8
Jar 2 = 4 x 5/8 = 20/8

If 1/8 OF A CUP is now poured from Jar 2 into Jar 1, then:

Jar 1 = 12/8 + 1/8 = 13/8
Jar 2 = 20/8 - 1/8 = 19/8

Their difference is 19/8 - 13/8 = 6/8 = 3/4 = 0.75

QUESTION 57.
Since I want to park for 14 hours, the total amount of 30 is decreased by 1 for it being more than 10 hours, and 4 x 0.5 or 2 for the additional 4 hours. Therefore the exact amount I need is 30 - 1 - 2 = 27.

To use the minimum number of coins to establish 27, I need to consider that a thingamajig is worth more than a whatchamacallit. Therefore I am going to use thingamajigs as my control variable; they'll accumulate faster, and therefore they'll be less of them to get us where we need to go.

Next, I'm going to divide my total by the value of a thingamajig to make my first try through things:

27 / 5 = 5 (with a remainder of 2)

So at most I'll have 5 thingamajigs. Since the remainder 2 is not a multiple of 3, then 5 is too many thingamajigs. So let's see:

Try 5 thingamajigs: 27 - (5 x 5) has a remainder of 2, 2 is not divisible by 3
Try 4 thingamajigs: 27 - (5 x 4) has a remainder of 7, 7 is not divisible by 3
Try 3 thingamajigs: 27 - (5 x 3) has a remainder of 12, 12 is divisible by 3!

So that means we have 3 thingamajigs and 4 whatchamacallits

.: 3 + 4 = 7 coins total

QUESTION 58.
Since triangles A, B, C, D, and E are congruent they are all the same triangles. Since the width of the rectangle is 6 then the midpoint of the width, 3, is the length of one of the legs of those triangles. Since the length of the rectangle is 8 then the midpoint of the length, 4, is the other length of one of the legs of those triangles.

The hypotenuse can be computed thus:

```
c^2 = a^2 + b^2
    = 3^2 + 4^2
    = 9 + 16
    = 25
.: c = 5
```

You may also know your Pythagorean Triples, as these are 3-4-5 right triangles!

We must count shared hypotenuses and shared legs separately as per the question.

That means each of A, B, C, D, E have perimeters of

3 + 4 + 5 = 12
.: 12 x 5 = 60

We're not done yet though, as there is still one more triangle that is encompassed by triangles, A, B, C and D (but not E). Its legs are 6 and 8. Its hypotenuse can be computed as 10, or, by using a multiple x 2 of your 3-4-5 Pythagorean Triple as 6-8-10. The perimeter of that triangle is 24.

Further furthermore, there is still another triangle: E in combo with the unnamed trapezoid in the upper right hand corner. It too has a perimeter of 24.

.: 60 + 24 + 24 = 108

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Answer Key to Sample Practice test for SHSAT GTSHSAT1 V1.0
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Answer key
1 B and 4:30pm
2 B
3 D
4 E and No
5 10
6 D
7 C
8 A
9 C
10 78
11 G
12 40
13 C
14 291.84
15 0
16 320
17 3
18 1200
19 D
20 2
21 555
22 D
23 A
24 H
25 0.25
26 B
27 C
28 B
29 B
30 E
31 45
32 E
33 14
34 12
35 120
36 84
37 22.5
38 27
39 A
40 9
41 -1.064
42 E
43 2.1
44 I
45 10
46 3
47 15
48 -1
49 -12
50 0
51 305
52 43088
53 36
54 12
55 30
56 0.75
57 7
58 108