

Hi, I'm Greg. I'm a NYC tutor! I love helping students. I tutor many subjects, assist with homework help, etc. I mainly specialize in specialized tests.

As it turns out, I haven't been able to get to do as many livestreams as I have in past years (yet, hopefully that changes). Therefore, I thought it would be fun to start a Problem Of The Day Series. I will put up a problem and leave it running for a while. You guys will then analyze it, and come up with possible solutions and alternative solutions on your own. I'll eventually post the answer in some manner.

For now we'll play it by ear how that will happen and for how long I'll leave up a problem. But right now I'm thinking of keeping the problem up maybe 2 hours minimum and maybe even in some cases 4 or 5 hours depending upon the dynamics and my situation. Unlike my AMA (Ask Me Anything) livestream sessions, I will not be checking in every few minutes although I may from time to time join into the discussion. Again, the idea is for you guys to discuss out the problem.

Please be respectful to each other in this endeavor and let's make this fun, educational and forward-thinking. Keep the comments within the spirit of what I'm doing here. Please email me at GregsTutoringNYC@gmail.com if needed.

HERE'S THE PROBLEM: <—
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If the bases of an isosceles trapezoid are doubled and its height is halved, what impact would that have on the area of the trapezoid?

- A) The area would always double
- B) The area would always be half
- C) The area would always increase 50%
- D) The area would always decrease 50%
- E) The area would always stay the same
- F) There is not enough information to determine what will always be the case
- G) There is not enough information to determine what will ever be the case

HERE'S THE SOLUTION:
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The area of a trapezoid, whether isosceles or not, is the average of its bases times it's height:

$$A = \frac{(b_1 + b_2)}{2} \times h$$

If the bases double and the height halves that give us:

$$A = \frac{2b_1 + 2b_2}{2} \times \frac{h}{2} = \frac{2(b_1 + b_2)}{2} \times \frac{h}{2}$$

That simplifies to:

$$A = \frac{(b_1 + b_2)}{2} \times h$$

which is the original equation. ∴ The area stays the same Choice E

By the way if there was a thought that the area halved, well, Choice B and Choice D both represent that, therefore those wouldn't be the case.

We could have also brute forced some sample numbers say a top base of 2 a bottom base of 4 and a height of 10. That would yield an area of $\frac{2 + 4}{2} \times 10 = 30$ units.

If we doubled the bases we'd get 4 and 8 and if we halved the height we'd get 5; the area in that case would be $6 \times 5 = 30$ units as well.

If you didn't know the direct formula to compute the area of a trapezoid, you could have also broken down the problem into three parts: a rectangle, with two right triangles adjacent to it. The left and right triangles are congruent as this was an isosceles trapezoid, but that would turn out neutral information depending upon how you solved this. Using "the normal formulas" we'd get:

$$\begin{aligned}A_{\text{initialrectangle}} &= l \times w = b_1 \times h \\A_{\text{initiallefttriangle}} &= bh/2 = ((b_2 - b_1)/2) \times h / 2 = h(b_2 - b_1)/4 \\A_{\text{initialrighttriangle}} &= bh/2 = ((b_2 - b_1)/2) \times h / 2 = h(b_2 - b_1)/4 \\A_{\text{initiallefttriangle}} + A_{\text{initialrighttriangle}} &= h(b_2 - b_1)/2 \\A_{\text{all}} &= b_1 \times h + h(b_2 - b_1)/2\end{aligned}$$

$$\begin{aligned}A_{\text{revisedrectangle}} &= l \times w \text{ .: for us: } 2b_1 \times h/2 = b_1 \times h \\A_{\text{revisedlefttriangle}} &= bh/2 \text{ .: for us: } (2(b_2 - b_1)/2) \times (h/2) / 2 = (b_2 - b_1) \times h / 4 \\A_{\text{revisedrighttriangle}} &= bh/2 \text{ .: for us: } (2(b_2 - b_1)/2) \times (h/2) / 2 = (b_2 - b_1) \times h / 4 \\A_{\text{revisedlefttriangle}} + A_{\text{revisedrighttriangle}} &= h(b_2 - b_1)/2 \\A_{\text{all}} &= b_1 \times h + h(b_2 - b_1)/2\end{aligned}$$

Or using test numbers with this approach:

$$\begin{aligned}A_{\text{initialrectangle}} &= 2 \times 10 = 20 \\A_{\text{initiallefttriangle}} &= (4-2)/2 = 1 \times h = 1 \times 10 = 10 / 2 = 5 \\A_{\text{initialrighttriangle}} &= 5 \\A_{\text{all}} &= 20 + 5 + 5 = 30\end{aligned}$$

$$\begin{aligned}A_{\text{revisedrectangle}} &= 2 \times 2 \times 10/2 = 4 \times 5 = 20 \\A_{\text{revisedlefttriangle}} &= 2 \times 4 - 2 \times 2 = 8 - 4 = 4 / 2 = 2 \times 10/2 = 10 / 2 = 5 \\A_{\text{revisedrighttriangle}} &= 5 \\A_{\text{all}} &= 20 + 5 + 5 = 30\end{aligned}$$

- Greg / GregsTutoringNYC@gmail.com LLAP ☺