1. (**D**) The base for the percentage is the weight for the first year, 75 million tons. The increase from the first year to the next year is 105 million tons.

$\frac{105 \text{ million}}{75 \text{ million}} = 1.4 \times 100\% = 140\%$, which is Option D.

2. (**J**) The equation whose graph is the dashed line is

$$y = {}^{-}2x + 2.$$

Therefore, the region above the line is the graph of the inequality

$$y > {}^{-}2x + 2.$$

The dashed line means that the line does not belong to the shaded region. So the shaded region is the graph of the inequality $y > {}^{-}2x + 2$ and not that of $y \geq {}^{-}2x + 2$.

3. (**D**) Jessenia's total income for last week was \$25 + \$20 = \$45. Divide her income last week by her usual weekly income to determine the ratio between them and then multiply by 100 to obtain the percent.

$\frac{45}{25} \times 100\% = 1.8 \times 100\% = 180\%$

4. (**G**) First, we need to find the scale factor d using M and M′.

$5d = 12$

$d = \frac{12}{5}$

If N = (x, y), and N′ = (24, 12), then:

$\frac{12}{5}x = 24$

$x = 24\left(\frac{5}{12}\right) = 10$

and $\frac{12}{5}y = 12$

$y = 12\left(\frac{5}{12}\right) = 5$

So, N = (10, 5).

5. (**C**) Although n is a negative number, its square is positive. Therefore, $n^2 > 4$. From this, we obtain $\frac{1}{n^2} < \frac{1}{4}$. (Note that in taking reciprocals, the inequality is reversed.) In addition, we know that $\frac{1}{n^2}$ is a positive number.

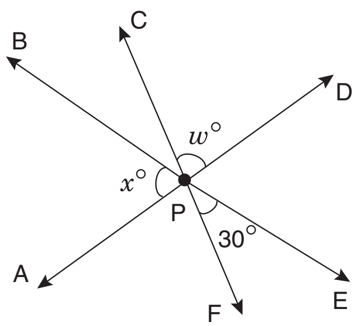
So $0 < \frac{1}{n^2} < \frac{1}{4}$ is the range of all possible values.

6. (**F**) Because triangles XWZ and YXZ are congruent, the corresponding angles WZX and XZY are equal. We know that $\overline{WZY}$ is a straight line segment, so angles WZX and XZY are supplementary (the sum of their measures is 180°). Because the angles are equal, the measure of each is equal to 90°. So Option F is correct and Options G and H are incorrect. Options J and K are not correct because angles XYZ and YXZ correspond to angles WXZ and XWZ, respectively. Because angle WZX is a right angle, angles XWZ and WXZ must each be smaller than angle WZX. So angle WZX cannot have the same measure as either angle XYZ or angle YXZ. Note that the diagram is deliberately not drawn to scale, so that the answer is not given away.

7. (**C**) $\frac{e^2 \bullet e^3}{e^6} = \frac{e^5}{e^6} = e^{5-6} = e^{{}^{-}1}$

8. (**J**) Because m and n are parallel lines, the two corresponding angles XWY and XVZ are equal. The measure of angle XWY is thus 50°. This means that the measure of angle XYW must be 180° − 60° − 50° = 70°.

9. (**A**) Label the diagram as follows:



Note that ∠BPC = ∠EPF. Therefore, the measure of ∠BPC is 30°. Since APD is a straight line, the measures of ∠APB, ∠BPC, and ∠CPD add up to 180°. Therefore:

$$x + 30 + w = 180$$
$$x + w = 180 - 30 = 150$$
$$w = 150 - x$$

10. (**J**) The sides of the triangle are equal in length, so the triangle is equilateral. The angles of an equilateral triangle are all equal to 60°.

$\overline{MPQ}$ is a straight line segment, so angle MPN and angle NPQ are supplementary. So the measure of angle NPQ is 180° − 60° = 120°.

11. (**E**) Translating P one unit to the right will increase the value of x by 1, so R = (3, 3). Reflecting R over the y-axis will keep the value of y the same, but change the sign on the value of x, so S = ($^-3$, 3). Finally, rotating S 90° clockwise about the origin will put the point back in the first quadrant and make T = (3, 3).

12. (**H**) The area of C_1 is πR^2 and that of C_2 is πr^2. We know that $\pi R^2 = 2\pi r^2$. Simplifying, we obtain $R^2 = 2r^2$. Taking the square root of each side, we obtain $R = \sqrt{2}\, r$.

13. (**E**) The point (n, n) is on the line and so must satisfy the equation. Substitute $x = n$ and $y = n$ and one obtains $n = \frac{1}{2}n + 2$, or $\frac{1}{2}n = 2$; therefore, $n = 4$.

14. (**K**) We are given that $x = {}^-6$, so we can immediately eliminate options Q and R. Q simplifies to x^3, which would result in a negative solution. Similarly, R simplifies to x, which would also result in a negative solution. Of the remaining options, the value of P would be between S and T, so we only need to evaluate S and T to find which is greater.

$\text{S} = x^2 + x = ({}^-6)^2 + ({}^-6) = 36 - 6 = 30$

$\text{T} = x^2 - x = ({}^-6)^2 - ({}^-6) = 36 + 6 = 42$

So, T has the greatest value.

15. (**D**) The sum of the interior angles of an octagon is (2 • 8 − 4) right angles, that is, 1,080°. Because all eight interior angles are equal, the measure of each interior angle is 1,080° ÷ 8 = 135°. Therefore, each exterior angle is 180° − 135° = 45°. The sum of all eight exterior angles is then 8 • 45° = 360°.

16. (**G**) $\overleftrightarrow{XYZ}$ is a straight line. So

$$90 + m + 30 + p = 180$$
$$m + p + 120 = 180$$
$$m + p = 60.$$

17. (**A**) List all the possibilities of drawing 5 balls out of 6 without replacement. Let R stand for a red ball and B stand for a blue ball. There are only 6 possibilities: RRRRR, RRRRB, RRRBR, RRBRR, RBRRR, BRRRR. Therefore, the probability that the 5 balls drawn are all red is $\frac{1}{6}$.

08-09

Answer Key for Grade 9 Mathematics

1. D	6. F	10. J	14. K
2. J	7. C	11. E	15. D
3. D	8. J	12. H	16. G
4. G	9. A	13. E	17. A
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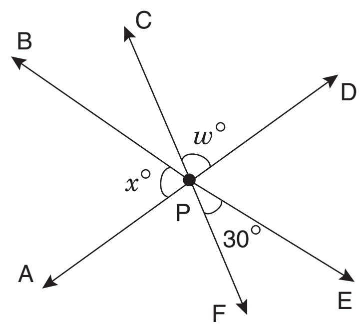

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09-10

Answer Key for Grade 9 Mathematics

1. D	6. F	10. J	14. K
2. J	7. C	11. E	15. D
3. D	8. J	12. H	16. G
4. G	9. A	13. E	17. A
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FOR GRADE 9 MATHEMATICS

1. (D) The base for the percentage is the weight for the first year (1960), 75 million tons. The increase from the first year to the ending year (1990) is 105 million tons (180 million − 75 million).

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FOR GRADE 9 MATHEMATICS

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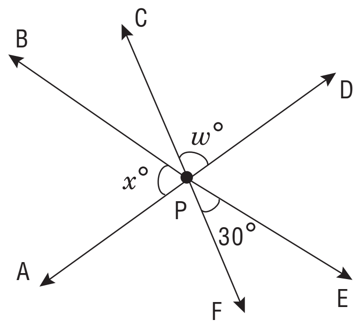

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17. (A) List all the possibilities of drawing 5 balls out of 6 without replacement. Let R stand for a red ball and B stand for a blue ball. There are only 6 possibilities: RRRRR, RRRRB, RRRBR, RRBRR, RBRRR, BRRRR. Therefore, the probability that the 5 balls drawn are all red is $\frac{1}{6}$.

10-11

Answer Key for Grade 9 Mathematics

1. D	6. F	10. J	14. K
2. J	7. C	11. E	15. D
3. D	8. J	12. H	16. G
4. G	9. A	13. E	17. A
5. C			

1. (A) First, cross-multiply to eliminate the denominators, and then solve for x:

$$4x = 3(3x - 15)$$
$$4x = 9x - 45$$
$$^{-}5x = {}^{-}45$$
$$x = 9$$

2. (G) Since P is on the x-axis, we know its y-value must equal 0. Use that in the equation to solve for x:

$$y = 15x - 45$$
$$0 = 15x - 45$$
$$45 = 15x$$
$$3 = x$$

So, the coordinates for P are (3, 0).

3. (D) In this case, the order in which you select the people is not important, so you cannot simply use the counting principle.

To solve this problem, first calculate the number of possible combinations for each gender.

Select 2 men from 4 men (a, b, c, d):

ab, ac, ad, bc, bd, cd

So, there are 6 ways to select 2 men from a group of 4 men.

Select 2 women from 5 women (v, w, x, y, z):

vw, vx, vy, vz, wx, wy, wz, xy, xz, yz

So, there are 10 ways to select 2 women from a group of 5 women.

The selection of one gender is independent of the selection of the other. Multiply the number of possible combinations for each gender: $6 \times 10 = 60$ different combinations.

4. (K) $\angle$QTS and $\angle$PTU are vertical angles, so they are congruent. Since $\angle$QRS is congruent to $\angle$QTS, then $\angle$QRS is also congruent to $\angle$PTU.

5. (A) Begin by finding a common base for each term. In this case, the common base is 2.

$$4 = 2^2$$
$$8 = 2^3$$

$$(4^3)(8^2) = (2^2)^3(2^3)^2$$
$$= (2^6)(2^6)$$
$$= 2^{12}$$
So, $x = 12$.

Alternatively, you could multiply the left side of the equation and then factor it:

$$(4^3)(8^2) = (4 \times 4 \times 4)(8 \times 8)$$
$$= (2 \times 2 \times 2 \times 2 \times 2 \times 2)\ (2 \times 2 \times 2 \times 2 \times 2 \times 2)$$
$$= 2^{12}$$

6. (G) Start with the original equation: $N = 1.\overline{25}$

Set up a second equation in which you multiply both sides of the original equation by a multiple of 10. You multiply by 10 for each digit in the repeating sequence. In this case, there are two digits, so you multiply by 10 twice, i.e., 100.

$$100N = 100(1.\overline{25})$$
$$100N = 125.\overline{25}$$

Now, subtract the two equations, then solve for N:

$$100N = 125.\overline{25}$$
$$-N = -1.\overline{25}$$
$$99N = 124$$
$$N = \frac{124}{99}$$

A shortcut is to recall that single-digit fractions with 9 as the denominator repeat, for example:

$$\frac{1}{9} = 0.\overline{1},\ \frac{2}{9} = 0.\overline{2}$$

This can be extended to two-digit fractions with 99 as the denominator, for example:

$$\frac{10}{99} = 0.\overline{10},\ \frac{20}{99} = 0.\overline{20}$$

In this case, $1.\overline{25} = 1\frac{25}{99} = \frac{124}{99}$

7. (**B**) Begin by converting from liters to quarts, and then from quarts to ounces. We know that 1 liter = 1.06 quarts, and 1 quart = 32 ounces, so:

1 liter = 1.06 × 32 = 33.92 ounces

We want to divide a 2-liter container of soda into 20-ounce containers.

2 liters = 2 × 33.92 = 67.84 ounces

67.84 ÷ 20 = 3.392 containers

The number 3.392 is greater than 3 but less than 4, so the answer is 3 full containers.

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8. (**K**) The equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept. Since the line passes through the origin, $b = 0$, so we only need to find the slope. Because we are given the point ($^-2$, 1) and the origin (0, 0), we can use the slope formula:

$$m = \frac{1 - 0}{^-2 - 0} = -\frac{1}{2}$$

Now, substitute the values for m and b in the equation:

$$y = mx + b$$

$$y = -\frac{1}{2}x + 0$$

$$y = -\frac{1}{2}x$$

---

9. (**C**) There are many ways to simplify this expression, but one way to begin is by simplifying the polynomial in the numerator:

$$\frac{6(2x^2 - 4x)}{3x}$$

$$= \frac{12x^2 - 24x}{3x}$$

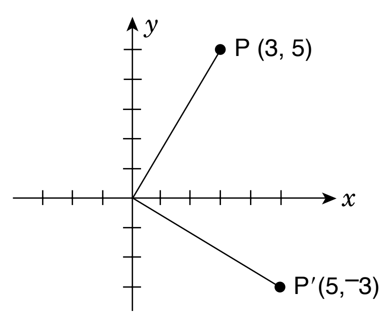
Divide the numerator and denominator by $3x$:

$$= 4x - 8$$

10. (**G**) If the coordinates of a point labeled R are (a, b), then a 90° **counterclockwise** rotation about the origin would make the coordinates of point R′ (^-b, a). A 90° **clockwise** rotation about the origin would make the coordinates of R′ (b, ^-a).

In the question, P is (3, 5) and P′ is (5, $^-3$). Using the rule stated above, P′ is the image after point P is rotated 90° clockwise.

Alternatively, it may help to make a sketch of this problem. Place the two points on the coordinate grid: Point P is in the first quadrant, and point P′ is in the fourth quadrant. Draw a line from each point to the origin. The angle formed at the origin should resemble a right angle, which is option G (90°).



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11. (**B**) First, simplify the inequality to get n on one side:

$$5 - n \geq 3n - 4$$

$$9 \geq 4n$$

$$\frac{9}{4} \geq n$$

$$2\frac{1}{4} \geq n$$

Since n is less than or equal to $2\frac{1}{4}$, the greatest integer value of n is 2.

12. (**H**) The volume of the cube is 729 cubic feet, so one side of that cube is $\sqrt[3]{729} = 9$ feet. The question asks for the length of an edge in inches.

9 feet × 12 = 108 inches

13. (**A**) Angle RNS is a right angle (90°). From the figure, we see that three smaller angles ($x°$, $y°$, and 47°) combine to make RNS:

$$x + y + 47 = 90$$
$$x + y = 43$$
$$y = 43 - x$$

14. (**F**) A common mistake on this type of problem is to treat a 150% increase as 1.5 times the original value. However, a 150% increase means adding 150% to the original value. If the original value is x, then x + 150% of $x = x + 1.5x = 2.5x$.

The present value is 2.5 times greater than the original value:

$$\$300{,}000 = 2.5x$$
$$\$120{,}000 = x$$

15. (**C**) Because QRS is a triangle, and the dashed line is a line of symmetry, the dashed line divides the triangle exactly in half and crosses side RS at its midpoint (7, 2).

To find the y-coordinate of S, note that the y-coordinate for R is 8 and the dashed line is at $y = 2$. The vertical distance between R and the line of symmetry is $8 - 2 = 6$. Subtract 6 from the y-value for the line of symmetry to find the y-coordinate of S: $2 - 6 = {}^{-}4$.

To find the x-coordinate of S, remember that RS must be a vertical line segment. Thus, the x-coordinate of S must be the same as the x-coordinate of R, which is 7.

So, the coordinates for S are (7, $^{-}4$).

16. (**J**) First, recognize that O and M represent the centers of the two circles. $\overline{OP}$ and $\overline{MP}$ are each a radius for one of the circles, and are given as length 20. Use the formula for the area of a circle to find the area of one-fourth of each circle:

$$\frac{1}{4}(20^2\pi) = 100\pi$$

The areas II + III and I + II each represent $\frac{1}{4}$ of a circle. So, II + III = 100π and I + II = 100π.

The area of square MNOP (20 × 20 = 400) is equivalent to I + II + III. Use the following formula to determine the area of region II:

Area of the square = (area of quarter circle M) + (area of quarter circle O) – (overlapping area)

$$\text{I} + \text{II} + \text{III} = (\text{I} + \text{II}) + (\text{II} + \text{III}) - \text{II}$$
$$400 = (100\pi) + (100\pi) - \text{II}$$
$$400 = 200\pi - \text{II}$$
$$\text{II} = 200\pi - 400$$

17. (**B**) Each triangle is a right triangle, and the angles formed at point Z are congruent because they are vertical angles. Thus, the two triangles are similar by definition. Set up the following proportion between similar sides to find x:

$$\frac{5}{3} = \frac{6}{x}$$

$$5x = 18$$

$$x = \frac{18}{5} = 3\frac{3}{5}$$

11-12

Answer Key for Grade 9 Mathematics

1. A	6. G	11. B	16. J
2. G	7. B	12. H	17. B
3. D	8. K	13. A	
4. K	9. C	14. F	
5. A	10. G	15. C	

1. (A) First, cross-multiply to eliminate the denominators, and then solve for x:

$$4x = 3(3x - 15)$$
$$4x = 9x - 45$$
$$^{-}5x = {}^{-}45$$
$$x = 9$$

2. (G) Since P is on the x-axis, we know its y-value must equal 0. Use that in the equation to solve for x:

$$y = 15x - 45$$
$$0 = 15x - 45$$
$$45 = 15x$$
$$3 = x$$

So, the coordinates for P are (3, 0).

3. (D) In this case, the order in which you select the people is not important, so you cannot simply use the counting principle.

To solve this problem, first calculate the number of possible combinations for each gender.

Select 2 men from 4 men (a, b, c, d):

ab, ac, ad, bc, bd, cd

So, there are 6 ways to select 2 men from a group of 4 men.

Select 2 women from 5 women (v, w, x, y, z):

vw, vx, vy, vz, wx, wy, wz, xy, xz, yz

So, there are 10 ways to select 2 women from a group of 5 women.

The selection of one gender is independent of the selection of the other. Multiply the number of possible combinations for each gender:
$6 \times 10 = 60$ different combinations.

4. (K) $\angle$QTS and $\angle$PTU are vertical angles, so they are congruent. Since $\angle$QRS is congruent to $\angle$QTS, then $\angle$QRS is also congruent to $\angle$PTU.

5. (A) Begin by finding a common base for each term. In this case, the common base is 2.

$$4 = 2^2$$
$$8 = 2^3$$

$$(4^3)(8^2) = (2^2)^3(2^3)^2$$
$$= (2^6)(2^6)$$
$$= 2^{12}$$
$$\text{So, } x = 12.$$

Alternatively, you could multiply the left side of the equation and then factor it:

$$(4^3)(8^2) = (4 \times 4 \times 4)(8 \times 8)$$
$$= (2 \times 2 \times 2 \times 2 \times 2 \times 2)(2 \times 2 \times 2 \times 2 \times 2 \times 2)$$
$$= 2^{12}$$

6. (G) Start with the original equation: $N = 1.\overline{25}$

Set up a second equation in which you multiply both sides of the original equation by a multiple of 10. You multiply by 10 for each digit in the repeating sequence. In this case, there are two digits, so you multiply by 10 twice, i.e., 100.

$$100N = 100(1.\overline{25})$$
$$100N = 125.\overline{25}$$

Now, subtract the two equations, then solve for N:

$$100N = 125.\overline{25}$$
$$-N = -1.\overline{25}$$
$$99N = 124$$
$$N = \frac{124}{99}$$

A shortcut is to recall that single-digit fractions with 9 as the denominator repeat, for example:

$$\frac{1}{9} = 0.\overline{1},\ \frac{2}{9} = 0.\overline{2}$$

This can be extended to two-digit fractions with 99 as the denominator, for example:

$$\frac{10}{99} = 0.\overline{10},\ \frac{20}{99} = 0.\overline{20}$$

In this case, $1.\overline{25} = 1\frac{25}{99} = \frac{124}{99}$

7. (**B**) Begin by converting from liters to quarts, and then from quarts to ounces. We know that 1 liter = 1.06 quarts, and 1 quart = 32 ounces, so:

1 liter = $1.06 \times 32 = 33.92$ ounces

We want to divide a 2-liter container of soda into 20-ounce containers.

2 liters = $2 \times 33.92 = 67.84$ ounces

$67.84 \div 20 = 3.392$ containers

The number 3.392 is greater than 3 but less than 4, so the answer is 3 full containers.

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8. (**K**) The equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept. Since the line passes through the origin, $b = 0$, so we only need to find the slope. Because we are given the point ($^-2$, 1) and the origin (0, 0), we can use the slope formula:

$$m = \frac{1 - 0}{^-2 - 0} = -\frac{1}{2}$$

Now, substitute the values for m and b in the equation:

$$y = mx + b$$

$$y = -\frac{1}{2}x + 0$$

$$y = -\frac{1}{2}x$$

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9. (**C**) There are many ways to simplify this expression, but one way to begin is by simplifying the polynomial in the numerator:

$$\frac{6(2x^2 - 4x)}{3x}$$

$$= \frac{12x^2 - 24x}{3x}$$

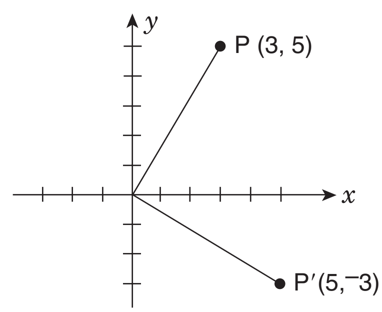
Divide the numerator and denominator by $3x$:

$$= 4x - 8$$

10. (**G**) If the coordinates of a point labeled R are (a, b), then a 90° **counterclockwise** rotation about the origin would make the coordinates of point R′ $(^-b, a)$. A 90° **clockwise** rotation about the origin would make the coordinates of R′ $(b, ^-a)$.

In the question, P is (3, 5) and P′ is (5, $^-3$). Using the rule stated above, P′ is the image after point P is rotated 90° clockwise.

Alternatively, it may help to make a sketch of this problem. Place the two points on the coordinate grid: Point P is in the first quadrant, and point P′ is in the fourth quadrant. Draw a line from each point to the origin. The angle formed at the origin should resemble a right angle, which is option G (90°).



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11. (**B**) First, simplify the inequality to get n on one side:

$$5 - n \geq 3n - 4$$

$$9 \geq 4n$$

$$\frac{9}{4} \geq n$$

$$2\frac{1}{4} \geq n$$

Since n is less than or equal to $2\frac{1}{4}$, the greatest integer value of n is 2.

12. (**H**) The volume of the cube is 729 cubic feet, so one side of that cube is $\sqrt[3]{729} = 9$ feet. The question asks for the length of an edge in inches.

9 feet × 12 = 108 inches

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13. (**A**) Angle RNS is a right angle (90°). From the figure, we see that three smaller angles ($x°$, $y°$, and 47°) combine to make RNS:

$$x + y + 47 = 90$$
$$x + y = 43$$
$$y = 43 - x$$

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14. (**F**) A common mistake on this type of problem is to treat a 150% increase as 1.5 times the original value. However, a 150% increase means adding 150% to the original value. If the original value is x, then x + 150% of $x = x + 1.5x = 2.5x$.

The present value is 2.5 times greater than the original value:

$$\$300{,}000 = 2.5x$$
$$\$120{,}000 = x$$

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15. (**C**) Because QRS is a triangle, and the dashed line is a line of symmetry, the dashed line divides the triangle exactly in half and crosses side RS at its midpoint (7, 2).

To find the y-coordinate of S, note that the y-coordinate for R is 8 and the dashed line is at $y = 2$. The vertical distance between R and the line of symmetry is $8 - 2 = 6$. Subtract 6 from the y-value for the line of symmetry to find the y-coordinate of S: $2 - 6 = {}^{-}4$.

To find the x-coordinate of S, remember that RS must be a vertical line segment. Thus, the x-coordinate of S must be the same as the x-coordinate of R, which is 7.

So, the coordinates for S are (7, $^{-}4$).

16. (**J**) First, recognize that O and M represent the centers of the two circles. $\overline{OP}$ and $\overline{MP}$ are each a radius for one of the circles, and are given as length 20. Use the formula for the area of a circle to find the area of one-fourth of each circle:

$$\frac{1}{4}(20^2\pi) = 100\pi$$

The areas II + III and I + II each represent $\frac{1}{4}$ of a circle. So, II + III = 100π and I + II = 100π.

The area of square MNOP (20 × 20 = 400) is equivalent to I + II + III. Use the following formula to determine the area of region II:

Area of the square = (area of quarter circle M) + (area of quarter circle O) – (overlapping area)

$$\text{I} + \text{II} + \text{III} = (\text{I} + \text{II}) + (\text{II} + \text{III}) - \text{II}$$
$$400 = (100\pi) + (100\pi) - \text{II}$$
$$400 = 200\pi - \text{II}$$
$$\text{II} = 200\pi - 400$$

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17. (**B**) Each triangle is a right triangle, and the angles formed at point Z are congruent because they are vertical angles. Thus, the two triangles are similar by definition. Set up the following proportion between similar sides to find x:

$$\frac{5}{3} = \frac{6}{x}$$
$$5x = 18$$
$$x = \frac{18}{5} = 3\frac{3}{5}$$

12-13

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1. (**C**) At the beginning (hour 0), the pool is empty. After 5 hours, the pool holds 2,000 gallons. Thus, the rate of change (or slope of the line) is $\frac{2{,}000 - 0}{5 - 0} = \frac{2{,}000}{5} = 400$ gallons per hour. To find the number of gallons after 20 hours, multiply the rate by the number of hours: $400 \times 20 = 8{,}000$ gallons.

2. (**H**) $\frac{2}{\left(\frac{4}{x}\right)} = \frac{3}{2}$

$2 \bullet \frac{x}{4} = \frac{3}{2}$

$2x = 6$

$x = 3$

3. (**D**) Since x is a negative number between $^-1$ and 0, assign a value to x in that range and calculate x^2. For example, let $x = -\frac{2}{3}$. Then $x^2 = \frac{4}{9}$, which roughly corresponds to point U.

4. (**J**) $7^{-2} + 7^{-1} + 7^0 = \frac{x}{49}$

$\frac{1}{49} + \frac{1}{7} + 1 = \frac{x}{49}$

$1 + 7 + 49 = x$

$57 = x$

5. (**B**) $\overline{WT}$ and $\overline{RV}$ are parallel, and $\overline{RT}$ is a transversal; thus ∠RVS and ∠TWS are alternate interior angles and are congruent. Angles WST and VSR are vertical angles, and therefore they are congruent. Since there are two sets of congruent angles, the third set of angles must also be congruent. Thus, △WTS is similar to △VRS. Since the triangles are similar, a proportion can be set up to solve for $\frac{WT}{ST}$.

$\frac{WT}{ST} = \frac{VR}{SR} = \frac{4}{5}$

6. (**H**) In order to add or subtract two numbers in scientific notation, the exponent on the 10 must be the same. Since the question asks for the value of $k \times 10^{19}$, change both terms into this same power of 10:

$12.6 \times 10^{18} = (1.26 \times 10) \times 10^{18} = 1.26 \times 10^{19}$

$1.1 \times 10^{17} = (0.011 \times 10^2) \times 10^{17} = 0.011 \times 10^{19}$

Now, perform the subtraction:

$(1.26 \times 10^{19}) - (0.011 \times 10^{19})$

$= (1.26 - 0.011) \times 10^{19}$

$= 1.249 \times 10^{19}$

Thus, $k = 1.249$

7. (**A**) The quickest way to solve this problem may be to test the options and see which results in the highest score. We can immediately eliminate options B, D, and E because those do not result in the correct transformation. Option A results in a score of $3 + 3 = 6$. Option C results in a score of $2 + 2 = 4$. Thus, A is the correct answer.

8. (**F**) First, determine the total number of pets that the students own by multiplying the number of pets owned by the number of students in each row of the table. Then add that column to get the total number of pets.

Number of Pets Owned	Number of Students	Number of Pets × Number of Students
0	5	0
1	7	7
2	3	6
3	4	12
4	0	0
5	1	5

Total: 30

Now, calculate the mean by dividing the total number of pets owned by the total number of students:

$\frac{30}{20} = 1\frac{1}{2}$

9. (**B**) Using the translation equation given in the question, set up two small equations to find n and r:

For n:
$x + 10 = 100$
$x = 90$

For r:
$y - 10 = 100$
$y = 110$

So, $(n, r) = (90, 110)$

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10. (**J**) First, calculate the volume of the cylinder:

$V = \pi r^2 h = \pi(4)^2(8) = 128\pi$ cubic inches

The volume of water in the cube will be the same as the volume of water in the full cylinder. Use the volume formula of a cube to calculate the depth (h) of the water in the cube:

$V = lwh$
$128\pi = (8)(8)h$
$128\pi = 64h$
$2\pi = h$

---

11. (**A**) Triangles MNR and TPR are similar, so use a proportion to solve for the length of $\overline{MN}$:

$\frac{MN}{MR} = \frac{TP}{TR}$

$\frac{MN}{2 + 12} = \frac{6}{12}$

$MN = \frac{1}{2}(14) = 7$ cm

12. (**H**) First, determine which integer values of x would make each inequality true:

$|x - 1| < 3$ can also be written as
$^-3 < x - 1 < 3$

Adding 1 to each term results in
$^-2 < x < 4$

Since these are only “less than” and not “less than or equal to,” the possible values of x for this inequality are $^-1$, 0, 1, 2, and 3.

Similarly, $|x + 2| < 4$ can also be written as
$^-4 < x + 2 < 4$

Subtracting 2 from each term results in
$^-2 < x < 2$

The possible values of x in this inequality are $^-1$, 0, and 1.

The possible x values in common between the two inequalities are $^-1$, 0, and 1, so the answer is 3.

13. (**B**) First, calculate the equation of the given line $f(x)$. The slope is calculated using the difference in y-values of two given points divided by the difference in x-values of those points. In this case, $m = \frac{(^{-}2 - 0)}{(0 - 1)} = 2$. The y-intercept (b) is the value of y when the line crosses the y-axis, so $b = {}^{-}2$. Using the slope-intercept form of a line ($y = mx + b$), the equation is $f(x) = 2x - 2$.

To determine which of the given values satisfies $y > f(x)$, or in this case $y > 2x - 2$, insert each value into the inequality to find which one makes the inequality true. You can immediately eliminate Option C, because the point (1, 0) is shown in the graph as one of the points on the line.

For inequality $y > 2x - 2$:

Option A
$^{-}7 > 2(^{-}2) - 2$
$^{-}7 > {}^{-}6$ is false

Option B
$^{-}3 > 2(^{-}1) - 2$
$^{-}3 > {}^{-}4$ is true

Option D
$1 > 2(2) - 2$
$1 > 2$ is false

Option E
$4 > 2(3) - 2$
$4 > 4$ is false
(Note that this point is also on the given line.)

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14. (**G**) A rational number is a number that can be written as a fraction. Since $p = q$, then $\frac{p}{q} = 1$, $\frac{p^2}{q^2} = 1$, and $p - q = 0$, all of which are rational. That leaves two expressions to test:

$$p + q = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

(irrational because $\sqrt{2}$ is irrational)

$$p^2 + q^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1 \text{ (rational)}$$

Thus, $p + q$ is not a rational expression.

15. (**C**) An x-intercept of 3 means the point (3, 0) is on line k. Using (3, 0) and ($^{-}3$, 4), calculate the slope (m) of the line:

$$m = \frac{(4-0)}{(^{-}3-3)} = \frac{4}{^{-}6} = -\frac{2}{3}$$

The equation of line k must contain slope $-\frac{2}{3}$, so only Options B and C are potentially correct.

Next, find which of the two equations is true for the point (3, 0). To solve, substitute 3 for x in each equation and find the one in which $y = 0$.

Option B: $y = -\frac{2}{3}(3) - 3 = {}^{-}2 - 3 = {}^{-}5$

Option C: $y = -\frac{2}{3}(3) + 2 = {}^{-}2 + 2 = 0$

Option C is the correct answer.

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16. (**G**) The question asks for the second integer, so let n be the second integer. Then, the sum of the 7 integers is:
$(n - 1) + n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) = 7k$
$7n + 14 = 7k$
$7(n + 2) = 7k$
$n + 2 = k$
$n = k - 2$

---

17. (**B**) Since the number of red flashes is known (15), first calculate where the robot would be after the 15 red flashes. For each red flash, $(x, y) \longrightarrow (x - 1, y + 4)$. So, after 15 red flashes: $(1 - [1 \times 15], {}^{-}2 + [4 \times 15]) = ({}^{-}14, 58)$

Next, use the point ($^{-}14$, 58) to calculate where the robot will be after n blue flashes. For each blue flash, $(x, y) \longrightarrow (x + 4, y - 5)$. So, after n blue flashes: $({}^{-}14 + 4n, 58 - 5n)$

The question states that the robot's final position is on the line $y = x$, which means the x- and y-coordinates will have the same value. To find n, set the two coordinates above as equal and solve for n:

$^{-}14 + 4n = 58 - 5n$
$9n = 72$
$n = 8$

Answer Key for Grade 9 Mathematics 13-14

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(Note that this point is also on the given line.)

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$$m = \frac{(4-0)}{(^{-}3-3)} = \frac{4}{^{-}6} = -\frac{2}{3}$$

The equation of line k must contain slope $-\frac{2}{3}$, so only Options B and C are potentially correct.

Next, find which of the two equations is true for the point (3, 0). To solve, substitute 3 for x in each equation and find the one in which $y = 0$.

Option B: $y = -\frac{2}{3}(3) - 3 = {}^{-}2 - 3 = {}^{-}5$

Option C: $y = -\frac{2}{3}(3) + 2 = {}^{-}2 + 2 = 0$

Option C is the correct answer.

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16. (**G**) The question asks for the second integer, so let n be the second integer. Then, the sum of the 7 integers is:
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17. (**B**) Since the number of red flashes is known (15), first calculate where the robot would be after the 15 red flashes. For each red flash, $(x, y) \longrightarrow (x - 1, y + 4)$. So, after 15 red flashes: $(1 - [1 \times 15], {}^{-}2 + [4 \times 15]) = ({}^{-}14, 58)$

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14-15

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Total: 30

Now, calculate the mean by dividing the total number of pets owned by the total number of students:

$$\frac{30}{20} = 1\frac{1}{2}$$

2. (**J**) Since x is a negative number between $^{-}1$ and 0, assign a value to x in that range and calculate x^2. For example, let $x = -\frac{2}{3}$. Then $x^2 = \frac{4}{9}$, which roughly corresponds to point U.

3. (**C**) $\frac{2}{(4/x)} = \frac{3}{2}$

$2 \bullet \frac{x}{4} = \frac{3}{2}$

$2x = 6$

$x = 3$

4. (**F**) Begin by finding a common base for each term. In this case, the common base is 2.

$$4 = 2^2$$
$$8 = 2^3$$

$$(4^3)(8^2) = (2^2)^3(2^3)^2$$
$$= (2^6)(2^6)$$
$$= 2^{12}$$

So, $x = 12$.

Alternatively, you could multiply the left side of the equation and then factor it:

$(4^3)(8^2) = (4 \times 4 \times 4)(8 \times 8)$

$= (2 \times 2 \times 2 \times 2 \times 2 \times 2)\ (2 \times 2 \times 2 \times 2 \times 2 \times 2)$

$= 2^{12}$

5. (**B**) Each triangle is a right triangle, and the angles formed at point Z are congruent because they are vertical angles. Thus, the two triangles are similar by definition. Set up the following proportion between similar sides to find x:

$\frac{5}{3} = \frac{6}{x}$

$5x = 18$

$x = \frac{18}{5} = 3\frac{3}{5}$

6. (**J**) First, calculate the volume of the cylinder:

$V = \pi r^2 h = \pi(4)^2(8) = 128\pi$ cubic inches

The volume of water in the cube will be the same as the volume of water in the full cylinder. Use the volume formula of a cube to calculate the depth (h) of the water in the cube:

$V = lwh$

$128\pi = (8)(8)h$

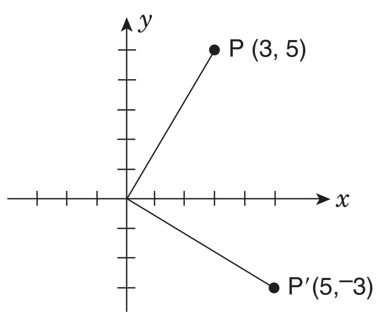
$128\pi = 64h$

$2\pi = h$

7. (B) If the coordinates of a point labeled R are (a, b), then a 90° counterclockwise rotation about the origin would make the coordinates of point R′ $(^{-}b, a)$. A 90° clockwise rotation about the origin would make the coordinates of R′ $(b, ^{-}a)$.

In the question, P is (3, 5) and P′ is $(5, ^{-}3)$. Using the rule stated above, P′ is the image after point P is rotated 90° clockwise.

Alternatively, it may help to make a sketch of this problem. Place the two points on the coordinate grid: Point P is in the first quadrant, and point P′ is in the fourth quadrant. Draw a line from each point to the origin. The angle formed at the origin should resemble a right angle, which is option B (90°).



8. (H) In order to add or subtract two numbers in scientific notation, the exponent on the 10 must be the same. Since the question asks for the value of $k \times 10^{19}$, change both terms into this same power of 10:

$12.6 \times 10^{18} = (1.26 \times 10) \times 10^{18} = 1.26 \times 10^{19}$

$1.1 \times 10^{17} = (0.011 \times 10^{2}) \times 10^{17} = 0.011 \times 10^{19}$

Now, perform the subtraction:

$(1.26 \times 10^{19}) - (0.011 \times 10^{19})$
$= (1.26 - 0.011) \times 10^{19}$
$= 1.249 \times 10^{19}$

Thus, $k = 1.249$

9. (C) At the beginning (hour 0), the pool is empty. After 5 hours, the pool holds 2,000 gallons. Thus, the rate of change (or slope of the line) is $\frac{2{,}000 - 0}{5 - 0} = \frac{2{,}000}{5} = 400$ gallons per hour.

To find the number of gallons after 20 hours, multiply the rate by the number of hours: $400 \times 20 = 8{,}000$ gallons.

10. (G) Using the translation equation given in the question, set up two small equations to find n and r:

For n:
$x + 10 = 100$
$x = 90$

For r:
$y - 10 = 100$
$y = 110$

So, $(n, r) = (90, 110)$

11. (A) Because both triangles are right triangles that share a vertex, they are similar. To find x, set up a proportion using the two known sides of each triangle:

$$\frac{(4 + x)}{1.0} = \frac{4}{0.8}$$

$$0.8\,(4 + x) = 1.0\,(4)$$

$$4 + x = 5$$

$$x = 1$$

12. (H) An x-intercept of 3 means the point (3, 0) is on line k. Using (3, 0) and $(^{-}3, 4)$, calculate the slope (m) of the line:

$$m = \frac{(4-0)}{(^{-}3-3)} = \frac{4}{6} = -\frac{2}{3}$$

The equation of line k must contain slope $-\frac{2}{3}$, so only Options G and H are potentially correct.

Next, find which of the two equations is true for the point (3, 0). To solve, substitute 3 for x in each equation and find the one in which $y = 0$.

Option G: $y = -\frac{2}{3}(3) - 3 = {}^{-}2 - 3 = {}^{-}5$

Option H: $y = -\frac{2}{3}(3) + 2 = {}^{-}2 + 2 = 0$

Option H is the correct answer.

---

13. (B) Since P is on the x-axis, we know its y-value must equal 0. Use that in the equation to solve for x:

$$y = 15x - 45$$
$$0 = 15x - 45$$
$$45 = 15x$$
$$3 = x$$

So, the coordinates for P are (3, 0).

---

14. (G) The question asks for the second integer, so let n be the second integer. Then, the sum of the 7 integers is:

$$(n - 1) + n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) = 7k$$
$$7n + 14 = 7k$$
$$7(n + 2) = 7k$$
$$n + 2 = k$$
$$n = k - 2$$

---

15. (B) A rational number is a number that can be written as a fraction. Since $p = q$, then $\frac{p}{q} = 1$, $\frac{p^2}{q^2} = 1$, and $p - q = 0$, all of which are rational. That leaves two expressions to test:

$$p + q = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

(irrational because $\sqrt{2}$ is irrational)

$$p^2 + q^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1 \text{ (rational)}$$

Thus, $p + q$ is not a rational expression.

---

16. (G) Since the number of red flashes is known (15), G calculate where the robot would be after the 15 red flashes. For each red flash, $(x, y) \longrightarrow (x - 1, y + 4)$. So, after 15 red flashes: $(1 - [1 \times 15], {}^{-}2 + [4 \times 15]) = ({}^{-}14, 58)$

Next, use the point $({}^{-}14, 58)$ to calculate where the robot will be after n blue flashes. For each blue flash, $(x, y) \longrightarrow (x + 4, y - 5)$. So, after n blue flashes: $({}^{-}14 + 4n, 58 - 5n)$

The question states that the robot's final position is on the line $y = x$, which means the x- and y-coordinates will have the same value. To find n, set the two coordinates above as equal and solve for n:

$${}^{-}14 + 4n = 58 - 5n$$
$$9n = 72$$
$$n = 8$$

---

17. (C) First, determine which integer values of x would make each inequality true:

$|x - 1| < 3$ can also be written as

${}^{-}3 < x - 1 < 3$

Adding 1 to each term results in ${}^{-}2 < x < 4$

Since these are only "less than" and not "less than or equal to," the possible values of x for this inequality are ${}^{-}1$, 0, 1, 2, and 3.

Similarly, $|x + 2| < 4$ can also be written as ${}^{-}4 < x + 2 < 4$

Subtracting 2 from each term results in ${}^{-}2 < x < 2$

The possible values of x in this inequality are ${}^{-}1$, 0, and 1.

The possible x values in common between the two inequalities are ${}^{-}1$, 0, and 1, so the answer is 3.

15-16

Answer Key for Grade 9 Mathematics

1. A	7. B	13. B
2. J	8. H	14. G
3. C	9. C	15. B
4. F	10. G	16. G
5. B	11. A	17. C
6. J	12. H	

1. (**B**) Each triangle is a right triangle, and the angles formed at point Z are congruent because they are vertical angles. Thus, the two triangles are similar by definition. Set up the following proportion between similar sides to find x:

$$\frac{5}{3} = \frac{6}{x}$$

$$5x = 18$$

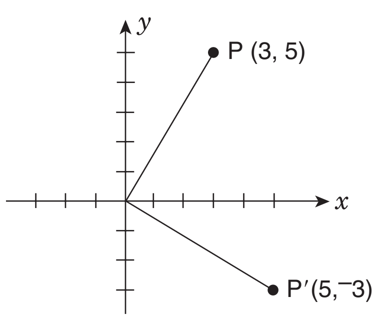
$$x = \frac{18}{5} = 3\frac{3}{5}$$

---

2. (**G**) If the coordinates of a point labeled R are (a, b), then a 90° counterclockwise rotation about the origin would make the coordinates of point R′ $(^-b, a)$. A 90° clockwise rotation about the origin would make the coordinates of R′ $(b, ^-a)$.

In the question, P is (3, 5) and P′ is (5, ⁻3). Using the rule stated above, P′ is the image after point P is rotated 90° clockwise.

Alternatively, it may help to make a sketch of this problem. Place the two points on the coordinate grid: Point P is in the first quadrant, and point P′ is in the fourth quadrant. Draw a line from each point to the origin. The angle formed at the origin should resemble a right angle, which is option G (90°).



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3. (**C**) At the beginning (hour 0), the pool is empty. After 5 hours, the pool holds 2,000 gallons. Thus, the rate of change (or slope of the line) is $\frac{2{,}000 - 0}{5 - 0} = \frac{2{,}000}{5} = 400$ gallons per hour.

To find the number of gallons after 20 hours, multiply the rate by the number of hours: $400 \times 20 = 8{,}000$ gallons.

4. (**F**) Begin by finding a common base for each term. In this case, the common base is 2.

$$4 = 2^2$$
$$8 = 2^3$$

$$(4^3)(8^2) = (2^2)^3(2^3)^2$$
$$= (2^6)(2^6)$$
$$= 2^{12}$$

So, $x = 12$.

Alternatively, you could multiply the left side of the equation and then factor it:

$(4^3)(8^2) = (4 \times 4 \times 4)(8 \times 8)$

$= (2 \times 2 \times 2 \times 2 \times 2 \times 2)(2 \times 2 \times 2 \times 2 \times 2 \times 2)$

$= 2^{12}$

---

5. (**B**) Since P is on the x-axis, we know its y-value must equal 0. Use that in the equation to solve for x:

$$y = 15x - 45$$
$$0 = 15x - 45$$
$$45 = 15x$$
$$3 = x$$

So, the coordinates for P are (3, 0).

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6. (**J**) Since x is a negative number between ⁻1 and 0, assign a value to x in that range and calculate x^2. For example, let $x = -\frac{2}{3}$. Then $x^2 = \frac{4}{9}$, which roughly corresponds to point U.

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Now, perform the subtraction:

$(1.26 \times 10^{19}) - (0.011 \times 10^{19})$

$= (1.26 - 0.011) \times 10^{19}$

$= 1.249 \times 10^{19}$

Thus, $k = 1.249$

8. (**F**) First, determine the total number of pets that the students own by multiplying the number of pets owned by the number of students in each row of the table. Then add that column to get the total number of pets.

Number of Pets Owned	Number of Students	Number of Pets × Number of Students
0	5	0
1	7	7
2	3	6
3	4	12
4	0	0
5	1	5

Total: 30

Now, calculate the mean by dividing the total number of pets owned by the total number of students:

$$\frac{30}{20} = 1\frac{1}{2}$$

9. (**C**) Since y is temperature and x is time, we can set up two points with the given information. The first point (0, 60) is when the oven is off. The second point (5, 350) indicates when the oven reaches the temperature of 350° which occurs after 5 minutes. Use these two points to find the slope (m) of the line:

$$m = \frac{350 - 60}{5 - 0} = \frac{290}{5} = 58$$

The first point (0, 60) indicates that the y-intercept (b) is 60.

Using slope-intercept form ($y = mx + b$), the equation is $y = 58x + 60$.

10. (**H**) First, determine which integer values of x would make each inequality true:

$|x - 1| < 3$ can also be written as

$^-3 < x - 1 < 3$

Add 1 to each term to simplifly the inequality
$^-2 < x < 4$

Since these are only "less than" and not "less than or equal to," the possible values of x for this inequality are $^-1$, 0, 1, 2, and 3.

Similarly, $|x + 2| < 4$ can also be written as
$^-4 < x + 2 < 4$

Subtract 2 from each term to simplifly the inequality
$^-2 < x < 2$

The possible values of x in this inequality are $^-1$, 0, and 1.

The possible x values in common between the two inequalities are $^-1$, 0, and 1, so the answer is 3.

11. (**B**) A rational number is a number that can be written as a fraction. Since $p = q$, then $\frac{p}{q} = 1$, $\frac{p^2}{q^2} = 1$, and $p - q = 0$, all of which are rational. That leaves two expressions to test:

$$p + q = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

(irrational because $\sqrt{2}$ is irrational)

$$p^2 + q^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1 \text{ (rational)}$$

Thus, $p + q$ is not a rational expression.

12. (**G**) Using the translation equation given in the question, set up two small equations to find n and r:

For n:
$x + 10 = 100$
$x = 90$

For r:
$y - 10 = 100$
$y = 110$

So, $(n, r) = (90, 110)$

13. (B) The question asks for the second integer, so let n be the second integer. Then, the sum of the 7 integers is:

$$(n - 1) + n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) = 7k$$

$$7n + 14 = 7k$$

$$7(n + 2) = 7k$$

$$n + 2 = k$$

$$n = k - 2$$

---

14. (H) Since $a \boxdot \frac{b}{c} = \frac{a}{\left(\frac{b}{c}\right)}$ then $2 \boxdot \frac{4}{x} = \frac{2}{\left(\frac{4}{x}\right)}$

$$\frac{2}{\left(\frac{4}{x}\right)} = \frac{3}{2}$$

$$4 = 3\left(\frac{4}{x}\right)$$

$$4 = \frac{12}{x}$$

$$4x = 12$$

$$x = 3$$

---

15. (D) First, calculate the volume of the cylinder:

$V = \pi r^2 h = \pi(4)^2(8) = 128\pi$ cubic inches

The volume of water in the cube will be the same as the volume of water in the full cylinder. Use the volume formula of a cube to calculate the depth (h) of the water in the cube:

$$V = lwh$$

$$128\pi = (8)(8)h$$

$$128\pi = 64h$$

$$2\pi = h$$

---

16. (F) Because both triangles are right triangles that share a vertex, they are similar. To find x, set up a proportion using the two known sides of each triangle:

$$\frac{(4 + x)}{1.0} = \frac{4}{0.8}$$

$$0.8\,(4 + x) = 4$$

$$4 + x = 5$$

$$x = 1$$

---

17. (C) An x-intercept of 3 means the point (3, 0) is on line k. Using (3, 0) and ($^-3$, 4), calculate the slope (m) of the line:

$$m = \frac{(4-0)}{(^-3-3)} = \frac{4}{^-6} = -\frac{2}{3}$$

The equation of line k must contain slope $-\frac{2}{3}$, so only Options B and C are potentially correct.

Next, find which of the two equations is true for the point (3, 0). To solve, substitute 3 for x in each equation and find the one in which $y = 0$.

Option B: $y = -\frac{2}{3}(3) - 3 = {}^-2 - 3 = {}^-5$

Option C: $y = -\frac{2}{3}(3) + 2 = {}^-2 + 2 = 0$

Option C is the correct answer.

16-17

Answer Key for Grade 9 Mathematics

1. B	7. C	13. B
2. G	8. F	14. H
3. C	9. C	15. D
4. F	10. H	16. F
5. B	11. B	17. C
6. J	12. G	

1. (12) $S(x)$ is the sum of all positive even integers less than or equal to x. 1, 2, 3, 4, 5, and 6 are all integers less than 7. Take the positive integers from the list and find the sum:

$S(7) = 2 + 4 + 6 = 12$

---

2. (56) $\sqrt{16} \bullet \sqrt{196} = 4 \bullet 14 = 56$

---

3. (B) When $\overline{MN}$ is translated 1 unit left, the distance between M′ and M is 1 unit, which is the base of the parallelogram. The height of the parallelogram is the vertical distance from M to N. Since M is at $y = 5$ and N is at $y = 1$, the height is $5 - 1 = 4$ units. The area of a parallelogram is base $\times$ height, so the area is $1 \times 4 = 4$ square units.

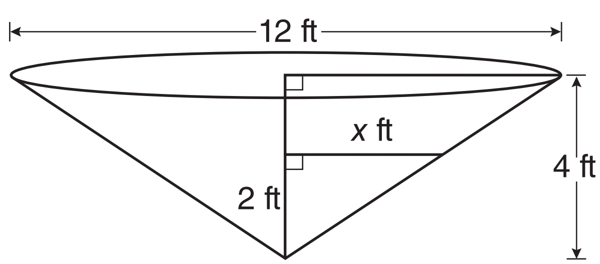
---

4. (H) $\dfrac{p^{12} \bullet p^0}{p^{-4}} = \left(p^{12} \bullet p^0\right) \dfrac{p^4}{1} = p^{(12+0+4)} =$

$p^{(12+4)} = p^{16}$

---

5. (A) First, find the radius when the depth of the water is 2 ft. Set up two similar right triangles as shown below:



Use a proportion to find x. Since the diameter of the right inverted cone is 12 ft, the radius is 6 ft:

$\dfrac{x}{6} = \dfrac{2}{4}$

$x = 3$ ft

Now, find the volume of the cone with a radius of 3 ft and a height of 2 ft:

$V = \dfrac{1}{3} r^2 \pi h = \dfrac{1}{3}(3^2)\pi(2) = 3\pi(2) = 6\pi$

---

6. (H) Use the slope formula to figure out the slope of line l.

Slope of line $l = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2b - b}{2a - a} = \dfrac{b}{a}$

---

7. (A) The line of best fit should be close to as many points as possible. In this case, very few of the points are on or next to the line. So, this is not a strong model for the data, because most of the points are not close to the line.

---

8. (G) Set up the two equations and subtract them from one another to find the price per hour:

$$\begin{aligned} y + 7x &= 420 \\ -\ y + 4x &= 270 \\ \hline 3x &= 150 \\ x &= 50 \end{aligned}$$

To find the fixed fee, use one of the equations $(y + 7x = 420$ or $y + 4x = 270)$ and solve for y, using $x = 50$.

$$\begin{aligned} y + 4x &= 270 \\ y + 4(50) &= 270 \\ y + 200 &= 270 \\ y &= 70 \end{aligned}$$

---

9. (D) Point R is at (4, 3). If (x, y) is rotated 180° about the origin: $R(x, y) \to (-x, -y)$. Therefore, $R(4, 3) \to (-4, -3)$.

---

10. (F)

$$\frac{15.3 \times 10^{-8}}{1.5 \times 10^4} = \left(\frac{15.3}{1.5}\right) \times \frac{10^{-8}}{10^4} = 10.2 \times \frac{10^{-8}}{10^4}$$

Then use the rule of exponents to simplify.

$10.2 \times 10^{(-8-4)} = 10.2 \times 10^{-12}$

Rewrite the answer so that it is standard scientific notation form.

1.02×10^{-11}

11. (C) Substitute the approximation $\pi = 3.14$ into each expression and solve to find which expression results in a negative value:

$4 - \pi = 0.86$

$3\pi - 9 = 0.42$

$12 - 4\pi = -0.56$

$36 - 9\pi = 7.74$

So, the answer is $12 - 4\pi$.

---

12. (F) Triangles NPQ and MPR are similar, so corresponding sides of the triangles are proportional. Set up a proportion to find $\overline{MR}$.

$$\frac{\overline{MR}}{\overline{MP}} = \frac{\overline{NQ}}{\overline{NP}}$$

$$\frac{\overline{MR}}{x+5} = \frac{10}{5}$$

$5(\overline{MR}) = 10(x + 5)$

$5(\overline{MR}) = 10x + 50$

$\overline{MR} = 2x + 10$

13. (C) Use the Pythagorean Theorem:

$x^2 + y^2 = 20^2$

Substitute $x = 2y$ into the equation and solve for y.

$(2y)^2 + y^2 = 20^2$

$4y^2 + y^2 = 400$

$5y^2 = 400$

$y^2 = 80$

$y = \sqrt{80}$

17-18

Answer Key for Grade 9 Mathematics

1. 12	6. H	11. C
2. 56	7. A	12. F
3. B	8. G	13. C
4. H	9. D	
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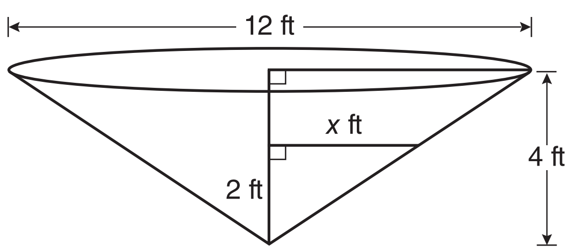

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$$\frac{\overline{MR}}{x+5} = \frac{10}{5}$$

$5(\overline{MR}) = 10(x + 5)$

$5(\overline{MR}) = 10x + 50$

$\overline{MR} = 2x + 10$

13. In this problem, $x = 3$, $y = 4$, and $z = 8$. Substitute those values into the given equation and simplify.

$$\frac{(3 \bullet 8) + (3 \bullet 4)}{2} + (8 \bullet 4) =$$

$$\frac{24 + 12}{2} + 32 =$$

$$\frac{36}{2} + 32 =$$

$$18 + 32 = 50$$

18-19

Answer Key for Grade 9 Mathematics

1. 12	6. H	11. C
2. 56	7. A	12. F
3. B	8. G	13. C
4. H	9. D	
5. A	10. F	

1. (–7) The function goes through points (0, 1) and (1, 3). Use those points to determine the equation of the function:

Slope: $\frac{3-1}{1-0} = \frac{2}{1} = 2$

It can be determined from the graph that the *y*-intercept is 1.

Equation: $y = 2x + 1$

Now plug in $x = {}^{-}4$ to find *y*:

$y = 2(-4) + 1 = -8 + 1 = -7$

2. (2) First, solve the second equation for *y*:

$x + 2y = 6$

$2y = 6 - x$ Apply the additive inverse property; subtract *x* from both sides of the equation

$y = \frac{6-x}{2}$ Apply the multiplicative inverse property; divide both sides of the equation by 2

Now set the two expressions for *y* equal to each other:

$\frac{3}{2}x - 1 = \frac{6-x}{2}$ Apply the multiplicative inverse property; multiply both sides by 2

$3x - 2 = 6 - x$ Apply the additive inverse property; add *x* to both sides of the equation

$4x - 2 = 6$ Apply the additive inverse property; add 2 to both sides of the equation

$4x = 8$ Apply the multiplicative inverse property; divide both sides of the equation by 4

$x = 2$

3. (10) The function first begins decreasing at (2, 10) and begins decreasing again at (12, 10). The difference in *x*-values is $12 - 2 = 10$.

4. (C) Let *x* represent the distance between the pole and the point where the wire attaches to the wall. Use the Pythagorean Theorem to find *x*:

$x^2 + 8^2 = 17^2$

$x^2 + 64 = 289$

$x^2 = 225$

$x = \sqrt{225} = 15$

5. (C) According to the scatter plot, as the potassium value increases, so does the nitrates value. Therefore, this is a positive association.

6. (C) In order to subtract the expressions, rewrite them so that they have the same exponent on the 10.

$(1.8 \times 10^6) - (7.3 \times 10^5)$

$= (1.8 \times 10^6) - (0.73 \times 10^6)$

$= (1.8 - 0.73) \times 10^6$

$= 1.07 \times 10^6$

7. (A) Rewrite the repeating decimals as fractions:

$x = 0.666666\ldots$ Let *x* equal the repeating decimal

$10x = 6.66666\ldots$ Multiply both sides of the equation by 10 to move the decimal one place to the right

$10x = 6.6666\ldots$ Subtract the two equations

$-x = -0.6666$

$9x = 6.0000\ldots$ Apply the multiplicative inverse property; divide both sides by 9

$x = \frac{6}{9} = \frac{2}{3}$ Simplify the fractions to lowest terms (if needed)

Perform the same process for $0.\overline{2}$

$10x = 2.2222\ldots$

$-x = -0.2222$

$9x = 2.0000\ldots$

$x = \frac{2}{9}$

Then multiply:

$\frac{2}{3} \times \frac{2}{9} = \frac{4}{27}$

8. (B) Solve for *x*:

$x = 4y - 2$

Since $x > 5$, then $4y - 2 > 5y > \frac{7}{4}$ or 1.75 since *y* is an integer, therefore the least possible integer value of *y* is 2

9. (B) The slope of the line of best fit is –3.25. Slope is $\frac{y}{x}$, or in this case, $\frac{\text{gas mileage}}{\text{engine size}}$. So, for every 1 L increase in engine size, the gas mileage decreases by 3.25 mpg.

10. (D) The problem gives two points: (4:00, 47) and (10:00, 32). Use that information to find the rate of change:

$$\frac{32 - 47}{10{:}00 - 4{:}00} = \frac{-15}{6} = \frac{-5}{2}$$

So, the temperature change was $-\frac{5}{2}$ ° F each hour.

To find the temperature at 2:00 a.m., which is two hours before 4:00 a.m., subtract $-\frac{5}{2}$ from 47 twice:

$$47 - 2\left(-\frac{5}{2}\right) = 47 + 5 = 52$$

Therefore, the temperature at 2:00 a.m. was 52° F.

11. (C) The new position of A (h, k) after rotating 90 degree will become A'(k, –h). Rotating 90° clockwise moves the line segment to the fourth quadrant. So, M' becomes (0, 1) and N' becomes (0, –1).

12. (D) Triangle RTS is a right triangle. First, find the lengths of the two legs (TS and RS). Then the Pythagorean Theorem can be used to find the length of $\overline{RT}$.

In rectangle STNM, TN is 2 cm, so SM is also 2 cm. Similarly, NM is 8 cm, so TS is also 8 cm.

In rectangle PRMQ, PQ is 10 cm, so RM is also 10 cm. Since $RM = RS + SM$, use the values of RM and SM to calculate the length of $\overline{RS}$, in centimeters:

$RS + SM = RM$

$RS + 2 = 10$

$RS = 8$

Now use the Pythagorean Theorem to find the length of:

$(RS)^2 + (TS)^2 = (RT)^2$

$8^2 + 8^2 = (RT)^2$

$64 + 64 = (RT)^2$

$128 = (RT)^2$

$\sqrt{128} = RT$

13. (D) In order to minimize the value of N, find the least possible, $(2x-1)^2$. Since this expression is squared, the least possible value is 0.

$(2x-1)^2 = 0$ Take the square root of both sides of the equation

$2x - 1 = 0$ Apply the additive inverse property; add 1 to both sides of the equation

$2x = 1$ Apply the multiplicative inverse property; divide both sides of the equation by 2

$x = \frac{1}{2}$

19-20

Answer Key for Grade 9 Mathematics

1. –7
2. 2
3. 10
4. C
5. C
6. C
7. A
8. B
9. B
10. D
11. C
12. D
13. D

1. (**−6**) Since the microchip is a square, the area of the microchip is $(1.2 \times 10^{-3})^2$ square meter.

$$(1.2 \times 10^{-3})^2 = (1.2)^2 \times (10^{-3})^2$$
$$= 1.44 \times 10^{-6}$$

So the value of a, the exponent, is −6.

---

2. (**66**) First, determine the number of people who do not use Soap L.

$$264 + 136 = 400$$

Then determine what percentage of those people use Soap M.

$$\frac{264}{400} = 0.66 = 66\%$$

Since 66% of the people who do not use Soap L use Soap M, the value of x is 66.

3. (**−5**) A linear function consists of ordered pairs that make a linear equation true, with a consistent slope, m, and a y-intercept, b. Use the slope formula and the two given ordered pairs to determine the slope.

$$m = \frac{2 - (-1)}{-4 - (-1)} = \frac{3}{-3} = -1$$

Then use the slope and one of the given ordered pairs to determine the y-intercept. The equation is in slope-intercept form.

$$y = (-1)x + b$$
$$2 = (-1)(-4) + b$$
$$2 = 4 + b$$
$$-2 = b$$

Use the slope and the y-intercept to determine the value of R. The equation is in slope-intercept form.

$$y = (-1)x + -2$$
$$R = (-1)(3) + -2$$
$$R = -3 + -2$$
$$R = -5$$

4. (**G**) Use the Pythagorean theorem, $A^2 + B^2 = C^2$, to find the distance between the two given points. A right triangle can be drawn in the coordinate system using the two given points as vertices.

To determine the lengths of the legs of the right triangle, find the absolute values of the difference between the *x*-coordinates and the difference between the *y*-coordinates.

$$|3 - 11| = 8$$

$$|20 - 5| = 15$$

Use the lengths of the legs, 8 units and 15 units, to determine the length of the hypotenuse, *h*, which is the distance, in units, between the two given points.

$$8^2 + 15^2 = h^2$$

$$64 + 225 = h^2$$

$$289 = h^2$$

$$\sqrt{289} = h$$

$$17 = h$$

The length of the hypotenuse is 17 units.

5. (**D**) Use properties of equations to successively transform the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ (where *a* and *b* are different numbers) results.

$$3(x - 4) + 4x = 4 - x + 8(6 + x)$$

$$3x - 12 + 4x = 4 - x + 48 + 8x$$

$$3x + 4x - 12 = 4 + 48 - x + 8x$$

$$7x - 12 = 52 + 7x$$

$$(7x - 7x) - 12 = 52 + (7x - 7x)$$

$$0 - 12 = 52 + 0$$

$$^{-}12 = 52$$

The simplest form of the given equation is $^{-}12 = 52$, which is not a true statement. Therefore, there is no solution to the given equation.

6. (**E**) Since rational numbers have a decimal expansion that terminates or repeats, determine the decimal expansion of the number in each option. The option that represents a number with a decimal expansion that terminates or repeats is a rational number.

Option E:

$\frac{3}{8} = 0.375$

Option F

$\pi = 3.14159...$

Option G

$\sqrt{3} = 1.73205...$

Option H

$\sqrt{83} = 9.11043...$

Option E has a decimal expansion that terminates; therefore, it is a rational number. The decimal expansions for the other options do not terminate or repeat.

7. (**B**) Use the properties of integer exponents to generate a numerical expression that is equivalent to the given expression.

$$\frac{6^{-10}}{6^2} = \frac{1}{6^2 \times 6^{10}} = \frac{1}{6^{12}}$$

The given expression is equivalent to $\frac{1}{6^{12}}$.

8. (**G**) Use the slope formula to determine the slope, *m,* between the ordered pairs in the table. If the slope between each pair of ordered pairs is the same, then the function is linear.

Option E:

Using the ordered pairs $(-1, 3)$ and $(-3, 5)$:

$$m = \frac{3-5}{-1-(-3)} = \frac{-2}{2} = -1$$

Using the ordered pairs $(0, 1)$ and $(-1, 3)$:

$$m = \frac{1-3}{0-(-1)} = \frac{-2}{1} = -2$$

Since the slopes do not match, this is not a linear function.

Option F:

Using the ordered pairs $(-2, 7)$ and $(-1, 4)$:

$$m = \frac{7-4}{-2-(-1)} = \frac{3}{-1} = -3$$

Using the ordered pairs $(-1, 4)$ and $(0, 3)$:

$$m = \frac{4-3}{-1-0} = \frac{1}{-1} = -1$$

Since the slopes do not match, this is not a linear function.

Option G:

Using the ordered pairs $(-4, -17)$ and $(-3, -12)$:

$$m = \frac{-17-(-12)}{-4-(-3)} = \frac{-5}{-1} = 5$$

Using the ordered pairs $(-3, -12)$ and $(1, 8)$:

$$m = \frac{-12-8}{-3-1} = \frac{-20}{-4} = 5$$

The slopes match, so this is a linear function.

Option H:

Using the ordered pairs $(-1, 3)$ and $(0, 3)$:

$$m = \frac{3-3}{-1-0} = \frac{0}{-1} = 0$$

Response continued on next page

Using the ordered pairs $(0, 3)$ and $(2, 5)$:

$$m = \frac{3 - 5}{0 - 2} = \frac{-2}{-2} = 1$$

Since the slopes do not match, this is not a linear function.

Option G is the only linear function and is the correct answer.

9. (**C**) First, translate both cell diameters to standard form.

$6 \times 10^{-7} = 0.0000006$

$3 \times 10^{-8} = 0.00000003$

Then use division to compare the two numbers.

$0.0000006 \div 0.00000003 = 20$

The diameter of Biological Cell A is 20 times the diameter of Biological Cell B.

10. (**F**) First, reorder the ordered pairs so that the numbers of seasons played, x, are increasing.

(1, 3), (3, 5), (4, 6), (6, 10)

Then examine the numbers of goals scored, y, to determine whether there is a pattern.

The pattern is that as x increases, so does y. This exemplifies a positive association between x, the number of seasons played, and y, the number of goals scored.

11. (**D**) Use properties of equations to determine the value of y in the given equation.

$$0.25(y + 8) = 15$$

$$0.25y + 2 = 15$$

$$0.25y = 13$$

$$\frac{0.25y}{0.25} = \frac{13}{0.25}$$

$$y = 52$$

The value of y in the given equation is 52.

12. (**H**) The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Since the diameter of the ball is 24 inches, the radius is half of that, or 12 inches.

$$V = \frac{4}{3}\pi(12)^3 = 2{,}304\pi$$

13. (**C**) Since 5 is the square root of 25, and 6 is the square root of 36, the values of *n* that satisfy the given condition are all the integers greater than 25 and less than 36. That set of integers is {26, 27, 28, 29, 30, 31, 32, 33, 34, 35}. There are 10 numbers in the set, so Option C is the correct answer.

20-21

Answer Key for Grade 9 Mathematics

1. –6	6. E	11. D
2. 66	7. B	12. H
3. –5	8. G	13. C
4. G	9. C	
5. D	10. F	

1. **(12)** $S(x)$ is the sum of all positive even integers less than or equal to x. 1, 2, 3, 4, 5, and 6 are all integers less than 7. Take the positive integers from the list and find the sum:

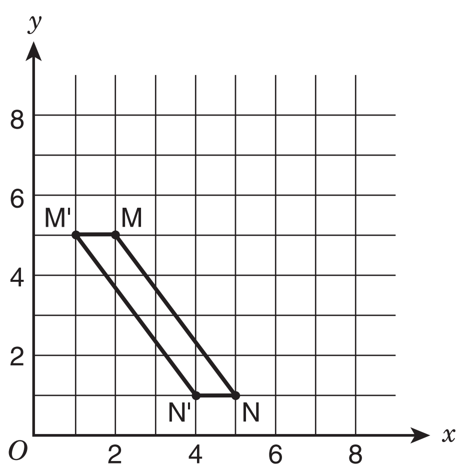
$S(7) = 2 + 4 + 6 = 12$

---

2. **(56)** $\sqrt{16} \cdot \sqrt{196} = 4 \cdot 14 = 56$

---

3. **(B)** When $\overline{MN}$ is translated 1 unit left, the distance between M′ and M is 1 unit, which is the base of the parallelogram. The height of the parallelogram is the vertical distance from M to N. Since M is at $y = 5$ and N is at $y = 1$, the height is $5 - 1 = 4$ units. The area of a parallelogram is base × height, so the area is $1 \times 4 = 4$ square units.

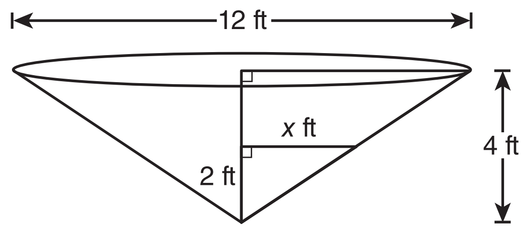


4. **(H)** $\dfrac{p^{12} \cdot p^{0}}{p^{-4}} = \left(p^{12} \cdot p^{0}\right)\dfrac{p^{4}}{1} =$

$p^{(12+0+4)} = p^{(12+4)} = p^{16}$

---

5. **(A)** Find the radius when the depth of the water is 2 ft. Set up two similar right triangles as shown below:



Use a proportion to find x. Since the diameter of the right inverted cone is 12 ft, the radius is 6 ft:

$$\frac{x}{6} = \frac{2}{4}$$

$x = 3$ ft

Find the volume of the cone with a radius of 3 ft and a height of 2 ft:

$$V = \frac{1}{3}r^2\pi h = \frac{1}{3}\left(3^2\right)\pi(2) = 3\pi(2) = 6\pi$$

6. **(H)** Use the slope formula to figure out the slope of line l.

Slope of line $l = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2b - b}{2a - a} = \frac{b}{a}$

7. **(A)** The question asks to describe the line of best fit for the graph. The line of best fit is a strong model when the most of the points are close to the line as possible. In this case, very few of the points are on or next to the line. Since there aren't many points close to the line, this is not a strong model for the data.

8. **(G)** Set up the two equations and subtract them from one another to find the price per hour:

$(y + 7x = 420) - (y + 4x = 270)$

$3x = 150$

$x = 50$

To find the fixed fee, use one of the equations $(y + 7x = 420$ or $y + 4x = 270)$ and solve for y, using $x = 50$.

$y + 4x = 270$
$y + 4(50) = 270$
$y + 200 = 270$
$y = 70$

9. **(D)** Point R is at $(4, 3)$. If (x, y) is rotated 180° about the origin:
$R(x, y) \to (-x, -y)$. Therefore,
$R(4, 3) \to (-4, -3)$.

10. **(F)** $\frac{15.3 \times 10^{-8}}{1.5 \times 10^{4}} = \left(\frac{15.3}{1.5}\right) \times \frac{10^{-8}}{10^{4}} =$

$10.2 \times \frac{10^{-8}}{10}$

Use the rule of exponents to simplify.

$10.2 \times 10^{(-8-4)} = 10.2 \times 10^{-12}$

Rewrite the answer so that it is standard scientific notation form.

1.02×10^{-11}

11. **(C)** Substitute the approximation $\pi = 3.14$ into each expression and solve to find which expression results in a negative value:

$4 - \pi = 0.86$
$3\pi - 9 = 0.42$
$12 - 4\pi = -0.56$
$36 - 9\pi = 7.74$

So, the answer is $12 - 4\pi$.

12. **(F)** Triangles NPQ and MPR are similar. Corresponding sides of the triangles are proportional. Set up a proportion to find $\overline{MR}$.

$$\frac{\overline{MR}}{\overline{MP}} = \frac{\overline{NQ}}{\overline{NP}}$$

$$\frac{\overline{MR}}{x+5} = \frac{10}{5}$$

$$5\left(\overline{MR}\right) = 10(x+5)$$

$$5\left(\overline{MR}\right) = 10x + 50$$

$$\overline{MR} = 2x + 10$$

13. **(C)** $x = 3$, $y = 4$, and $z = 8$. Substitute those values into the equation and simplify.

$$\frac{(3 \cdot 8) + (3 \cdot 4)}{2} + (8 \cdot 4) =$$

$$\frac{24 + 12}{2} + 32 =$$

$$\frac{36}{2} + 32 =$$

$$18 + 32 = 50$$

21-22

Answer Key for Grade 9 Mathematics

1. 12	6. H	11. C
2. 56	7. A	12. F
3. B	8. G	13. C
4. H	9. D	
5. A	10. F	

1. **(11)** To determine the value of x, isolate x on one side of the equation:

$$0.44 = \frac{x}{25}$$

$$25(0.44) = \left(\frac{x}{25}\right)25$$

$$11 = x$$

2. **(3)** To determine the y-value of the point of intersection of the graphs of $y = 6x - 5$ and $y = {}^{-}3x + 7$, first set the y-values equal:

$$6x - 5 = ({}^{-}3x) + 7$$

$$9x - 5 = 7$$

$$9x = 12$$

$$x = \frac{4}{3}$$

Substitute the x-value of $\frac{4}{3}$ into either of the given equations to determine the value of y:

$$y = 6\left(\frac{4}{3}\right) - 5 \text{ or } y = ({}^{-}3)\left(\frac{4}{3}\right) + 7$$

Solve for y:

$$y = \left(\frac{24}{3}\right) - 5 = 8 - 5 = 3$$

or

$$y = \left(\frac{{}^{-}12}{3}\right) + 7 = ({}^{-}4) + 7 = 3$$

The value of y, 3, is the y-coordinate of the point of intersection.

3. **(20)** The function $y = 2x - 4$ is linear because it is in the form $y = mx + b$.

Determine the value of y using the value of x from $(12, y)$, 12, in the equation of the function $y = 2x - 4$:

$y = 2(12) - 4$
$y = 24 - 4$
$y = 20$

---

4. **(G)** 57 is a number located between perfect squares 49 and 64. The value of $\sqrt{49}$ is located at 7 on a number line. The value of $\sqrt{64}$ is located at 8 on a number line. Therefore, the value of $\sqrt{57}$ is located between 7 and 8 on a number line.

5. **(C)** Angle T (47°) and the adjacent angle to its immediate right (x) are supplementary, so the sum of the two angle measures is 180°:

$47° + x = 180°$

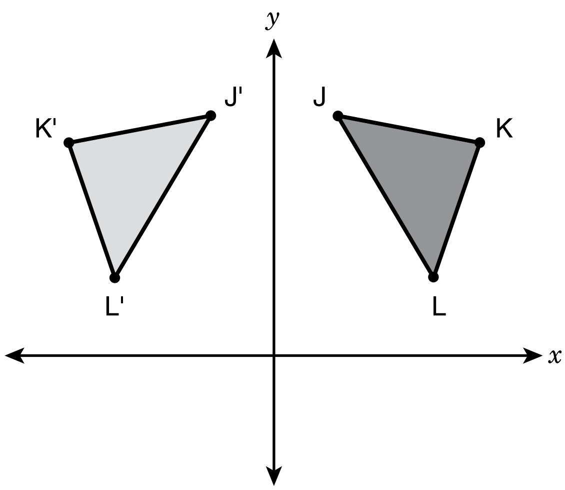
$x = 133°$

Because line k and line m are parallel and are both cut by line p, a transversal, corresponding angles are congruent. The 133° angle and angle J are corresponding angles, so the measure of angle J is 133°.

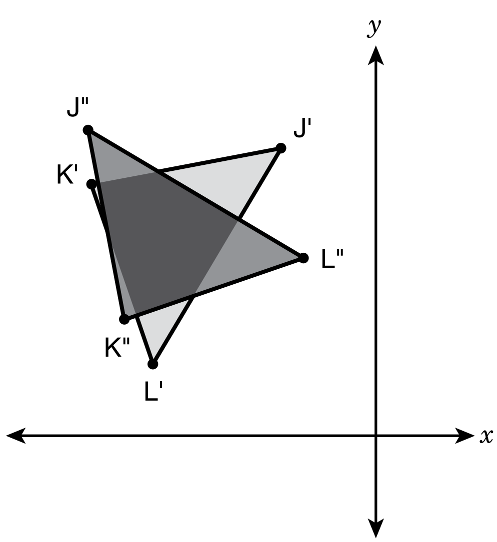
---

6. **(H)** $\sqrt{7}$ is an irrational number because it has a value of $\pm 2.645751311\ldots$, which is a non-repeating, non-terminating decimal, and non-repeating, non-terminating decimals are irrational numbers.

7. **(D)** First, reflect JKL over the *y*-axis to get J′K′L′, as seen by the reversed vertices.



Then, rotate J′K′L′ 90° counterclockwise about the center, since the order of the vertices is still the same but rotated.



8. **(F)** The line $y = x$ has a positive slope. The data points shown on the scatter plot trend in a positive direction, and the location of $y = x$ results in 5 of the data points above the line and 5 of the data points below the line. This is the most accurate line of best fit for this data.

9. **(A)** To determine the point of intersection of the graphs of the lines, set the equations equal:

$^{-}4x + 3 = 2x + 5$

Solve for x:

$^{-}6x = 2$

$x = \frac{2}{^{-}6}$

$x = -\frac{1}{3}$

This is the x-coordinate of the point of intersection. Use it to determine the y-coordinate by substituting $\left(-\frac{1}{3}\right)$ for x in either of the equations:

$y = 2\left(-\frac{1}{3}\right) + 5$ or $y = {}^{-}4\left(-\frac{1}{3}\right) + 3$

Solve either equation for y:

$y = \left(-\frac{2}{3}\right) + 5 = 4\frac{1}{3}$ or
$y = \frac{4}{3} + 3 = 4\frac{1}{3}$

The x-coordinate of the point of intersection is the value of x $\left(-\frac{1}{3}\right)$ and the y-coordinate of the point of intersection is the value of y $\left(4\frac{1}{3}\right)$, making the point of intersection of the graphs of the two lines $\left(-\frac{1}{3}, 4\frac{1}{3}\right)$.

10. **(G)** The rate of change is equal to the slope. Determine the rate of change for the linear function represented in the table:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(40 - 30)}{(24 - 12)} = \frac{10}{12} = \frac{5}{6}$$

The slope of a linear equation in the form $y = mx + b$ is the value of m, so the slope, or rate of change, of $y = 4x + 3$ is 4.

The rate of change of $y = 4x + 3$ is greater because $4 > \frac{5}{6}$.

---

11. **(A)** No solutions are possible because an absolute value can never be negative.

12. **(G)** The data have clustering because they form two distinct groups: one group (cluster) of data points is located in the area between (0, 0) and (1, 2), and the other group (cluster) of data points is located in the area between (4, 4) and (6, 5).

---

13. **(C)** In a right triangle, the sum of the squares of the leg lengths is equal to the square of the length of the hypotenuse. In the given figure, the leg lengths are 8 in. and 8 in., and the hypotenuse is x. Substitute the values into the equation $a^2 + b^2 = c^2$:

$$(8)^2 + (8)^2 = x^2$$

$$64 + 64 = x^2$$

$$x^2 = 128$$

$$x = \sqrt{128}$$

22-23

Answer Key for Grade 9 Mathematics

1. 11	6. H	11. A
2. 3	7. D	12. G
3. 20	8. F	13. C
4. G	9. A	
5. C	10. G	

Grade 9

Mathematics Explanations of Correct Answers

1. (**$^-7$**) The function goes through points (0, 1) and (1, 3). Use those points to determine the equation of the function:

Slope: $\frac{3 - 1}{1 - 0} = \frac{2}{1} = 2$

It can be determined from the graph that the y-intercept is 1.

Equation: $y = 2x + 1$

Now plug in $x = {}^-4$ to find y:

$y = 2({}^-4) + 1 = {}^-8 + 1 = {}^-7$

---

2. **(10)** The function first begins decreasing at (2, 10) and begins decreasing again at (12, 10). The difference in x-values is $12 - 2 = 10$.

3. **(2)** First, solve the second equation for y:

$x + 2y = 6$	
$2y = 6 - x$	Apply the additive inverse property; subtract x from both sides of the equation.
$y = \frac{6 - x}{2}$	Apply the multiplicative inverse property; divide both sides of the equation by 2.

Now set the two expressions for y equal to each other:

$\frac{3}{2}x - 1 = \frac{6 - x}{2}$	Apply the multiplicative inverse property; multiply both sides by 2.
$3x - 2 = 6 - x$	Apply the additive inverse property; add x to both sides of the equation.
$4x - 2 = 6$	Apply the additive inverse property; add 2 to both sides of the equation.
$4x = 8$	Apply the multiplicative inverse property; divide both sides of the equation by 4.
$x = 2$	

4. **(G)** Let x represent the distance between the pole and the point where the wire attaches to the wall. Use the Pythagorean theorem to find x:

$$x^2 + 8^2 = 17^2$$
$$x^2 + 64 = 289$$
$$x^2 = 225$$
$$x = \sqrt{225} = 15$$

---

5. **(C)** According to the scatter plot, as the potassium value increases, so does the nitrates value. Therefore, this is a positive association.

---

6. **(G)** In order to subtract the expressions, rewrite them so that they have the same exponent on the 10.

$$(1.8 \times 10^6) - (7.3 \times 10^5) =$$
$$(1.8 \times 10^6) - (0.73 \times 10^6) =$$
$$(1.8 - 0.73) \times 10^6 =$$
$$1.07 \times 10^6$$

7. **(A)** Rewrite the repeating decimals as fractions:

$x = 0.666666\ldots$	Let x equal the repeating decimal.
$10x = 6.66666\ldots$	Multiply both sides of the equation by 10 to move the decimal one place to the right.
$10x = 6.6666\ldots$ $\underline{-x = -0.6666\ldots}$	Subtract the two equations.
$9x = 6.0000\ldots$	Apply the multiplicative inverse property; divide both sides by 9.
$x = \frac{6}{9} = \frac{2}{3}$	Simplify the fraction to lowest terms (if needed).

Perform the same process for $0.\overline{2}$:

$$10x = 2.2222\ldots$$
$$\underline{-x = -0.2222\ldots}$$
$$9x = 2.0000\ldots$$
$$x = \frac{2}{9}$$

Then multiply:

$$\frac{2}{3} \times \frac{2}{9} = \frac{4}{27}$$

8. **(F)** Solve for x:

$$x = 4y - 2$$

Since $x > 5$, then $4y - 2 > 5$. So, $y > \frac{7}{4}$ or 1.75. Since y is an integer, the least possible integer value of y is 2.

---

9. **(B)** The slope of the line of best fit is -3.25. Slope is $\frac{\text{change in } y}{\text{change in } x}$,

or in this case, $\frac{\text{change in gas mileage}}{\text{change in engine size}}$.

So, for every 1 L increase in engine size, the gas mileage decreases by 3.25 mpg.

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10. **(H)** The problem gives two points: (4:00, 47) and (10:00, 32). Use that information to find the rate of change:

$$\frac{32 - 47}{10 - 4} = \frac{-15}{6} = \frac{-5}{2}$$

So, the temperature change was $-\frac{5}{2}$° F each hour.

To find the temperature at 2:00 a.m., which is 2 hours before 4:00 a.m., subtract $-\frac{5}{2}$ from 47 twice:

$$47 - 2\left(-\frac{5}{2}\right) = 47 + 5 = 52$$

Therefore, the temperature at 2:00 a.m. was 52° F.

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11. **(C)** The new position of $A(k, h)$ after rotating 90° clockwise will be $A'(k, -h)$. Rotating 90° clockwise moves the line segment to the fourth quadrant. So, M' becomes $(1, 0)$ and N' becomes $(0, -1)$.

12. **(H)** Triangle RTS is a right triangle. First, find the lengths of the two legs (TS and RS). Then the Pythagorean theorem can be used to find the length of $\overline{RT}$.

In rectangle STNM, TN is 2 cm, so SM is also 2 cm. Similarly, NM is 8 cm, so TS is also 8 cm.

In rectangle PRMQ, PQ is 10 cm, so RM is also 10 cm. Since RM = RS + SM, use the values of RM and SM to calculate the length of $\overline{RS}$, in centimeters:

$$RS + SM = RM$$
$$RS + 2 = 10$$
$$RS = 8$$

Now use the Pythagorean theorem to find the length of $\overline{RT}$:

$$(RS)^2 + (TS)^2 = (RT)^2$$
$$8^2 + 8^2 = (RT)^2$$
$$64 + 64 = (RT)^2$$
$$128 = (RT)^2$$
$$\sqrt{128} = RT$$

13. **(D)** In order to minimize the value of *N*, find the least possible value of $(2x - 1)^2$. Since this expression is squared, the least possible value is 0.

$(2x - 1)^2 = 0$	Take the square root of both sides of the equation.
$2x - 1 = 0$	Apply the additive inverse property; add 1 to both sides of the equation.
$2x = 1$	Apply the multiplicative inverse property; divide both sides of the equation by 2.
$x = \frac{1}{2}$	

Answer Key for Grade 9 Mathematics

1. -7
2. 10
3. 2
4. G
5. C
6. G
7. A
8. F
9. B
10. H
11. C
12. H
13. D

Grade 9
Mathematics Explanations of Correct Answers

1. **($^-6$)** Since the microchip is a square, the area of the microchip is $(1.2 \times 10^{-3})^2$ square meter.

$$(1.2 \times 10^{-3})^2 =$$
$$(1.2)^2 \times (10^{-3})^2 =$$
$$1.44 \times 10^{-6}$$

So the value of a, the exponent, is $^-6$.

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2. **(66)** First, determine the number of people who do not use Soap L.

$$264 + 136 = 400$$

Then determine what percentage of those people use Soap M.

$$\frac{264}{400} = 0.66 = 66\%$$

Since 66% of the people who do not use Soap L use Soap M, the value of x is 66.

3. **($^-5$)** A linear function consists of ordered pairs that make a linear equation true, with a consistent slope, m, and a y-intercept, b. Use the slope formula and the two given ordered pairs to determine the slope.

$$m = \frac{2 - (^-1)}{^-4 - (^-1)} = \frac{3}{^-3} = {^-1}$$

Then use the slope and one of the given ordered pairs to determine the y-intercept. The equation is in slope-intercept form.

$$y = (^-1)x + b$$
$$2 = (^-1)(^-4) + b$$
$$2 = 4 + b$$
$$^-2 = b$$

Use the slope and the y-intercept to determine the value of R. The equation is in slope-intercept form.

$$y = (^-1)x + {^-2}$$
$$R = (^-1)(3) + {^-2}$$
$$R = {^-3} + {^-2}$$
$$R = {^-5}$$

4. **(G)** Use the Pythagorean theorem, $A^2 + B^2 = C^2$, to find the distance between the two given points. A right triangle can be drawn in the coordinate system, using the two given points as vertices.

To determine the lengths of the legs of the right triangle, find the absolute values of the difference between the *x*-coordinates and the difference between the *y*-coordinates:

$|3 - 11| = 8$

$|20 - 5| = 15$

Use the lengths of the legs, 8 units and 15 units, to determine the length of the hypotenuse, *h*, which is the distance, in units, between the two given points:

$8^2 + 15^2 = h^2$

$64 + 225 = h^2$

$289 = h^2$

$\sqrt{289} = h$

$17 = h$

The length of the hypotenuse is 17 units.

5. **(C)** First simplify the equation:

$3(x - 4) + 4x = 4 - x + 8(6 + x)$

$3x - 12 + 4x = 4 - x + 48 + 8x$

$3x + 4x - 12 = 4 + 48 - x + 8x$

$7x - 12 = 52 + 7x$

$(7x - 7x) - 12 = 52 + (7x - 7x)$

$0 - 12 = 52 + 0$

$^-12 = 52$

Since $^-12 = 52$ is not a true statement, there is no solution to the given equation.

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6. **(E)** Since rational numbers have a decimal expansion that terminates or repeats, determine the decimal expansion of the number in each option. The option that represents a number with a decimal expansion that terminates or repeats is a rational number.

<u>Option E:</u>

$\frac{3}{8} = 0.375$

Option F

$\pi = 3.14159...$

Option G

$\sqrt{3} = 1.73205...$

Option H

$\sqrt{83} = 9.11043...$

Option E has a decimal expansion that terminates; therefore, it is a rational number. The decimal expansions for the other options do not terminate or repeat.

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7. **(B)** Use the properties of integer exponents to generate a numerical expression that is equivalent to the given expression:

$$\frac{6^{-10}}{6^2} = \frac{1}{6^2 \times 6^{10}} = \frac{1}{6^{12}}$$

The given expression is equivalent to $\frac{1}{6^{12}}$.

8. **(G)** Use the slope formula to determine the slope, *m,* between the ordered pairs in the table. If the slope between each pair of ordered pairs is the same, then the function is linear.

Using the ordered pairs $(-4, -17)$ and $(-3, -12)$:

$$m = \frac{-17 - (-12)}{-4 - (-3)} = \frac{-5}{-1} = 5$$

Using the ordered pairs $(-3, -12)$ and $(1, 8)$:

$$m = \frac{-12 - 8}{-3 - 1} = \frac{-20}{-4} = 5$$

The slopes match, so this is a linear function.

So the table in Option G is the one that represents a linear function.

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9. **(C)** First, translate both cell diameters to standard form:

$$6 \times 10^{-7} = 0.0000006$$

$$3 \times 10^{-8} = 0.00000003$$

Then, use division to compare the two numbers:

$0.0000006 \div 0.00000003 = 20$

The diameter of Biological Cell A is 20 times the diameter of Biological Cell B.

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10. **(F)** First, reorder the ordered pairs so that the numbers of seasons played, x, are increasing:

(1, 3), (3, 5), (4, 6), (6, 10)

Then examine the numbers of goals scored, y, to determine whether there is a pattern.

The pattern is that as x increases, so does y. This exemplifies a positive association between x, the number of seasons played, and y, the number of goals scored.

11. **(D)** Use properties of equations to determine the value of y in the given equation:

$0.25(y + 8) = 15$

$0.25y + 2 = 15$

$0.25y = 13$

$\frac{0.25y}{0.25} = \frac{13}{0.25}$

$y = 52$

12. **(H)** The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

The diameter of the ball is 24 inches, so the radius, r, is 12 inches.

$$V = \frac{4}{3}\pi(12)^3 = 2{,}304\pi$$

13. **(C)** Since 5 is the square root of 25, and 6 is the square root of 36, the values of *n* that satisfy the given condition are all the integers greater than 25 and less than 36. That set of integers is {26, 27, 28, 29, 30, 31, 32, 33, 34, 35}. There are 10 numbers in the set, so Option C is the correct answer.

Answer Key for Grade 9 Mathematics

1. -6	6. E	11. D
2. 66	7. B	12. H
3. -5	8. G	13. C
4. G	9. C	
5. C	10. F	